SECTION – A

1. The area of the region $\left\{(x,y): x^2 \le y \le 8-x^2, y \le 7\right\}$ is.

(1) 24 (2) 21 (3) 20 (4) 18

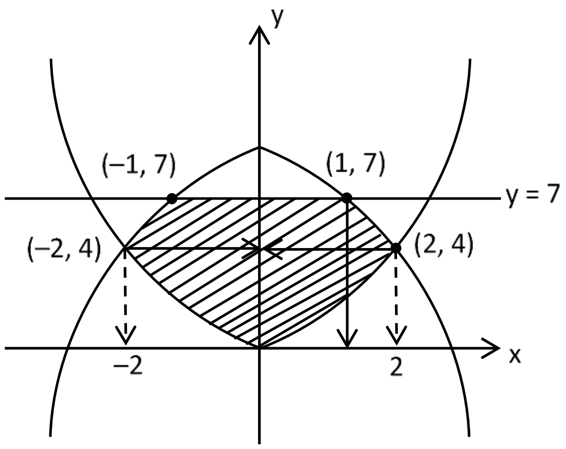
Sol. **(3)**

$y \ge x^2 \quad y \le 8-x^2 \qquad y \le 7$

$x^2 = 8 - x^2$

$x^2 = 4$

$x = \pm 2$



$$2\left(1.7 + \int_1^2 \left(8-2x^2\right)dx\right) - 2\int_0^1 \left(x^2\right)dx$$

$$= 2\left[7 + \left(8x - \frac{2x^3}{3}\right)_1^2\right] - 2\left(\frac{x^3}{3}\right)_0^1$$

$$= 2\left[7 + \left(16 - \frac{16}{3}\right) - \left(8 - \frac{2}{3}\right)\right] - 2\left(\frac{1}{3}\right)$$

$$= 2\left[7 + \frac{32}{3} - \frac{22}{3}\right] = 2\left[7 + \frac{10}{3}\right]\frac{-2}{3}$$

$$= \frac{60}{3} = 20$$

2. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$. If $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then 2a+ b -3c – 4d equal to

(1) 2004 (2) 2007 (3) 2005 (4) 2006

Sol. **(3)**

$Q = PAP^T$

$P^T. Q^{2007}. P = P^T. Q.Q \dots Q.P$

$= P^T(PAP^T)\,(P.AP^T) \dots (PAP^T)P.$

$\Rightarrow (P^TP)A(P^TP)A \dots A(P^TP)$

$$P^T.P = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}\begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore$ $P^T. Q^{2007}. P = A^{2007}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

a = 1, b = 2007, c = 0, d = 1

2a + b – 3c – 4d = 2 + 2007 – 4 = 2005

3. Negation of $(p \to q) \to (q \to p)$ is

(1) $(-q) \wedge p$ (2) $p \vee (\sim q)$ (3) $(\sim p) \vee q$ (4) $q \wedge (\sim p)$

Sol. **(4)**

$(p \to q) \to (q \to p)$

$\sim[\sim\ p \to q\ \wedge q \to p]$

$\Rightarrow\ p \to q\ \wedge \sim\ q \to p$

$\Rightarrow\ \sim p \vee q\ \wedge\ q \wedge \sim p$

$\Rightarrow q \wedge \sim p$.

4. Let C(α, β) be the circumcenter of the triangle formed by the lines

4x + 3y = 69,

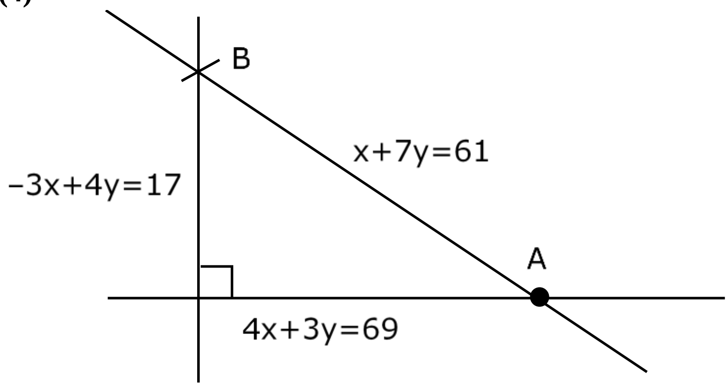
4y – 3x = 17 and

x + 7y = 61.

Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

(1) 18 (2) 15 (3) 16 (4) 17

Sol. **(4)**



4x + 28y = 244

4x + 3y = 69

– – –

25y = 175

y = 7, x = 12

A(12, 7)

$-3x + 4y = 17$

$3x + 21y = 183$

$25y = 200$

$y = 8, x = 5$

B(5, 8)

$\therefore$ Circumcenter

$\alpha = \frac{17}{2} \beta = \frac{15}{2}$

$\left(\frac{17}{2}, \frac{15}{2}\right)$

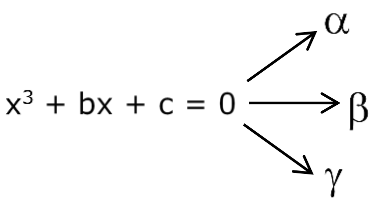
$(\alpha - \beta)^2 + \alpha + \beta$

$1+16 = 17$

5. Let α, β, γ, be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to

(1) $\frac{155}{8}$ (2) 21 (3) 19 (4) $\frac{169}{8}$

Sol. **(3)**



$\beta\gamma = 1$

$\alpha = -1$

Put $\alpha = -1$

$-1 - b + c = 0$

$c - b = 1$

also

$\alpha.\beta.\gamma = -c$

$-1 = -c \Rightarrow c = 1$

$\therefore b = 0$

$x^3 + 1 = 0$

$\alpha = -1, \beta = -w, \gamma = -w^2$

$\therefore b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$

$0 + 2 + 3 + 6 + 8 = 19$

6. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is:

(1) 752 (2) 772 (3) 782 (4) 792

Sol. **(4)**

$n(A \times B) = 10$

$^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 = 792$

7. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is

(1) 5481 (2) 3654 (3) 2436 (4) 1817

Sol. **(2)**

$\frac{^nC_r}{^nC_{r-1}} = 5 \qquad \frac{^nC_{r+1}}{^nC_r} = 4$

$\frac{n - r + 1}{r} = 5 \qquad n = 5r + 4 \ \ldots(2)$

$n = 6r - 1 \ldots (1)$

$\therefore n = 29, r = 5$

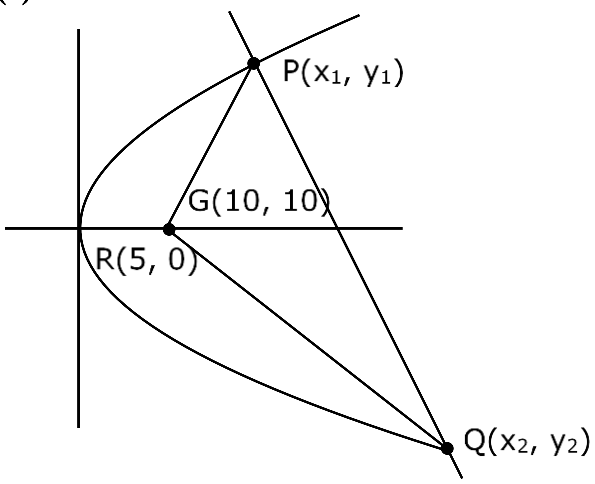
Coeff of 4th term = $^{29}C_3$

= 3654

8. Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q. Let the point G(10, 10) be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is

(1) 325 (2) 346 (3) 296 (4) 317

Sol. **(1)**



$y^2 = 20x$, $y = mx + c$

$y^2 = 20\left(\frac{y - c}{m}\right)$

$y^2 - \frac{20y}{m} + \frac{20c}{m} = 0 \qquad \frac{y_1 + y_2 + y_3}{3} = 10$

$\frac{20}{m} = 30$

$m = 2/3$

and $c - m = 6$

$c = \frac{2}{3} + 6 \Rightarrow \frac{20}{3} = c$

$y^2 - 30y + \frac{20 \times 20/3}{2/3} = 0 \Rightarrow \qquad y^2 - 30y + 200 = 0$

y = 10, y = 20

y = 20, x = 20 P(5, 10); (20, 20)Q

$\frac{20+5+x}{3} = 10 \Rightarrow x = 5$ $PQ^2 = 15^2 + 10^2 = 225 + 100 = 325$

9. Let $S_K \frac{1+2+...+K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}$ $(Bn^2 + Cn+D)$, where A, B, C, D $\in$N and A has least value. Then

(1) A + B is divisible by D

(2) A + B = 5 (D –C)

(3) A + C + D is not divisible by B

(4) A + B + D is divisible by 5

Sol. **(1)**

$S_k = \frac{k+1}{2}$

$S_k^2 = \frac{k^2+1+2k}{4}$

$\therefore \sum_{j-1}^{n} S_j^2 = \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n + n(n+1)\right]$

$= \frac{n}{4}\left[\frac{(n+1)(2n+1)}{6} + 1 + n + 1\right]$

$= \frac{n}{4}\left[\frac{2n^2+3n+1}{6} + n + 2\right]$

$= \frac{n}{4}\left[\frac{2n^2+9n+13}{6}\right] = \frac{n}{24}\left[2n^2+9n+13\right]$

A = 24, B = 2, C = 9, D = 13

10. The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is

(1) $2\sqrt{6}$

(2) $3\sqrt{6}$

(3) $6\sqrt{3}$

(4) $6\sqrt{2}$

Sol. **(2)**

$S_d = \left|\frac{(\vec{a}-\vec{b})\times(\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|}\right|$

$\bar{a} = (4,-2,-3)$

$\bar{b} = (1,3,4)$

$\bar{n}_1 = (4,5,3)$

$\bar{n}_2 = (3,4,2)$

$\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = \hat{i}(-2) - \hat{j}(-1) + \hat{k}(1) = (-2,1,1)$

$S_d = \frac{(3,-5,-7)\cdot(-2,1,1)}{\sqrt{6}} = \left|\frac{-6-5-7}{\sqrt{6}}\right| = 3\sqrt{6}$

11. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is.

(1) 16800 (2) 14800 (3) 18000 (4) 33600

Sol. **(1)**

IEEEE,

NNN, DD, P, C

$$\frac{8!}{3!2!}\times\frac{6!}{41}=16800$$

12. If the points with position vectors $\alpha\hat{i}+10\hat{j}+13\hat{k}, 6\hat{i}+11\hat{j}+11\hat{k}, \frac{9}{2}\hat{i}+\beta\hat{j}-8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to

(1) 49 (2) 36 (3) 25 (4) 16

Sol. **(2)**

$$(\alpha,10,13);(6,11,11),\left(\frac{9}{2},\beta,-8\right)$$

$$\frac{\alpha-6}{3/2}=\frac{-1}{11-\beta}=\frac{2}{19}$$

$$\alpha-6=\frac{3}{19} \qquad -19=22-2\beta$$

$$\alpha=6+\frac{3}{19}=\frac{117}{19} \qquad 2\beta=41$$

$$\therefore(19\alpha-6\beta)^2=(117-123)^2=36$$

13. In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random form the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is.

(1) $\frac{5}{14}$ (2) $\frac{3}{7}$ (3) $\frac{9}{28}$ (4) $\frac{2}{7}$

Sol. **(1)**

$P(A) = \frac{2}{10}$ $P(B) = \frac{3}{10}$ $P(C) = \frac{5}{10}$

$P(\text{Defective}/A) = \frac{3}{100}$, $P\left(\text{Defective}/B\right)=\frac{4}{100}$, $P\left(\text{Defective}/C\right)=\frac{2}{100}$

$$P(E) = \frac{5/10\times2/100}{\frac{2}{10}\times\frac{3}{100}+\frac{3}{10}\times\frac{4}{100}+\frac{5}{10}\times\frac{2}{100}}=\frac{10}{6+12+10}$$

$$=\frac{10}{28}$$

$$=\frac{5}{14}$$

14. If for $z = \alpha + i\beta$, $|z + 2| = z + 4(1 + i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation

(1) $x^2 + 3x - 4 = 0$ (2) $x^2 + 7x + 12 = 0$

(3) $x^2 + x - 12 = 0$ (4) $x^2 + 2x - 3 = 0$

Sol. **(2)**

$|z + 2| = |\alpha + i\beta + 2|$

$= \alpha + i\beta + 4 + 4i$

$\sqrt{(\alpha+2)^2+\beta^2} = (\alpha+4)+i(\beta+4)$ $\quad\quad$ $\beta + 4 = 0$

$(\alpha+2)^2+16=(\alpha+4)^2$ $\quad\quad$ $\beta = -4$

$\alpha^2 + 4 + 4\alpha + 16 = \alpha^2 + 16 + 8\alpha$

$4 = 4\alpha$

$\alpha = 1$

$\alpha = 1, \beta = -4$

$\alpha + \beta = -3, \alpha\beta = -4$

Sum of roots = – 7

Product of roots = 12

$x^2 + 7x + 12 = 0$

15. $\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1)^5}\right)\right)$ is equal to ______

(1) 24 (2) 9 (3) 18 (4) 15

Sol. **(3)**

$$\lim_{x\to 0}\left[\frac{1-\cos^2 3x}{9x^2}\right]\frac{9x^2}{\cos^3 4x}\cdot\frac{\left(\frac{\sin 4x}{4x}\right)^3\times 64x^3}{\left[\frac{\ln(1+2x)}{2x}\right]^5\times 32x^5}$$

$$\lim_{x\to 0}2\left(\frac{1}{2}\times\frac{9}{1}\times\frac{1\times 64}{1\times 32}\right)=18$$

16. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is

(1) $7(720)^2$ (2) 720 (3) $7(360)^2$ (4) $126(5!)^2$

Sol. **(4)**

$6!\times {}^7C_5 \times 5!$

$\Rightarrow 720 \times 21 \times 120$

$\Rightarrow 2 \times 360 \times 7 \times 3 \times 120$

$\Rightarrow 126 \times (5!)^2$

17. Let $f(x)=\frac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}, x\in[0.\pi]-\left\{\frac{\pi}{4}\right\}$. Then $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ is equal to

(1) $\frac{-2}{3}$ (2) $\frac{2}{9}$ (3) $\frac{-1}{3\sqrt{3}}$ (4) $\frac{2}{3\sqrt{3}}$

Sol. **(2)**

$$f(x) = -\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

$$f'(x) = -\frac{1}{2}\sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

$$f''(x) = -\sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right)\cdot\tan\left(\frac{x}{2}\frac{-\pi}{8}\right)\cdot\frac{1}{2}$$

$$f\left(\frac{7\pi}{12}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

$$f''\left(\frac{7\pi}{12}\right) = -\frac{1}{2}\sec^2\frac{\pi}{6}.\tan\frac{\pi}{6} = \frac{-1}{2}\cdot\frac{4}{3}\times\frac{1}{\sqrt{3}} = \frac{-2}{3\sqrt{3}}$$

$$f\left(\frac{7\pi}{12}\right)\cdot f''\left(\frac{7\pi}{12}\right) = \frac{2}{9}$$

18. If the eqation of the plane containing the line x + 2y + 3z – 4 = 0 2x + y –z + 5 and perpendicular to the plane $\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ is ax+by +cz = 4, then (a-b+c) is equal to

(1) 22 (2) 24 (3) 20 (4) 18

Sol. **(1)**

D.R's of line $\vec{n}_1 = -5\hat{i} + 7\hat{j} - 3\hat{k}$

D.R's of normal of second plane

$\vec{n}_2 = 5\hat{i} - 2\hat{j} - 3\hat{k}$

$\vec{n}_1 \times \vec{n}_2 = -27\hat{i} - 30\hat{j} - 25\hat{k}$

A point on the required plane is $\left(0, -\frac{11}{5}, \frac{14}{5}\right)$

The equation of required plane is

27x + 30y + 25z = 4

$\therefore$ a – b + c = 22

19. Let A = $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If |adj(adj(adj2A))| =$(16)^n$, then n is equal to

(1) 8 (2) 9 (3) 12 (4) 10

Sol. **(4)**

|A| = 2[3] – 1[2] = 4

$\therefore \left|adj\left(adj\left(adj2A\right)\right)\right|$

$= |2A|^{(n-1)^3} \Rightarrow |2A|^8 = 16^n$

$\Rightarrow \left(2^3|A|\right)^8 = 16^n$

$\Rightarrow \left(2^3 \times 2^2\right)^8 = 16^n$

$= 2^{40} = 16^n$

$= 16^{10} = 16^n \Rightarrow n = 10$

20. Let $I(x) = \int \frac{(x+1)}{x(1+x\,e^x)^2}\,dx$, x > 0. If $\lim_{x\to\infty} I(x) = 0$, then I(1) is equal to

(1) $\frac{e+1}{e+2} - \log_e(e+1)$

(2) $\frac{e+2}{e+1} + \log_e(e+1)$

(3) $\frac{e+2}{e+1} - \log_e(e+1)$

(4) $\frac{e+1}{e+2} + \log_e(e+1)$

Sol.

$$I(x) = \int \frac{(x+1)}{x(1+xe^x)^2}dx$$

$1 + xe^x = t$

$(xe^x + e^x)dx = dt$

$(x+1)dx = \frac{1}{e^x}dt$

$$\therefore \int \frac{dt}{xe^x.t^2} \Rightarrow \int \frac{dt}{(t-1)t^2} \Rightarrow \int \frac{dt}{t(t-1)\cdot t} \Rightarrow \int \frac{t-(t-1)}{t(t-1)t}dt$$

$$\Rightarrow \int \frac{dt}{t(t-1)} - \int \frac{dt}{t^2} \Rightarrow \int \frac{t-(t-1)}{t(t-1)}dt + \frac{1}{t} + c$$

$$\Rightarrow \ln|t-1| - \ln|t| + \frac{1}{t} + c$$

$$\Rightarrow \ln|xe^x| - \ln|1+xe^x| + \frac{1}{1+xe^x} + c$$

$$I(x) = \ln\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + c$$

$$\lim_{x\to\infty} I(x) = c = 0$$

$$\therefore I(1) = \ln\left|\frac{e}{1+e}\right| + \frac{1}{1+e}$$

$$= \ln e - \ln(1+e) + \frac{1}{1+e}$$

$$= \frac{e+2}{e+1} - \ln(1+e)$$

SECTION - B

21. Let A = {0,3 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R ={x,y) $\in$ A × A: x – y is odd positive integer or x – y = 2}. The minimum number of elements that must be aadded to the relation R, so that it is a symmetric relation, is equal to ______.

Sol. **(19)**

A = {0, 3, 4, 6, 7, 8, 9, 10}   3, 7, 9 $\rightarrow$ odd

$R = \{x - y = odd + ve \text{ or } x - y = 2\}$ 0, 4, 6, 8, 10 $\rightarrow$ even

$^3C_1 \cdot {}^5C_1 = 15 +$ (6, 4), (8, 6), (10, 8), (9, 7)

Min^m ordered pairs to be added must be

: 15 + 4 = 19

22. Let (t) denote the greatest integer $\le$ t, If the constant term in the expansion of $\left(3x^2 - \frac{1}{2x^5}\right)^7$ is α, then [α] is equal to ______.

Sol. **(1275)**

$$\left(3x^2 - \frac{1}{2x^5}\right)^7$$

$$T_{r+1} = {}^7C_r \left(3x^2\right)^{7-r} \left(-\frac{1}{2x^5}\right)^r$$

14 – 2r – 5r = 14 – 7r = 0

$\therefore r = 2$

$$\therefore T_3 = {}^7C_2 . 3^5 \left(-\frac{1}{2}\right)^2 = \frac{21 \times 243}{4} = 1275.75$$

$$\therefore [\alpha] = 1275$$

23. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and (–2, 0, 1) are at equal distance from the plane 2x + 3y – 6z + 7 = 0. If $\lambda_1 > \lambda_2$, then the distance of the point ($\lambda_1 - \lambda_2$, λ_2, λ_1) from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is ______.

Sol. **9**

2x + 3y – 6z + 7 = 0 $\left(\frac{5}{2}, 1, \lambda\right), (-2, 0, 1)$

$$d_1 = \left|\frac{5 + 3 - 6\lambda + 7}{7}\right| = d_2 = \left|\frac{-4 - 6 + 7}{7}\right|$$

$\Rightarrow |15 - 6\lambda| = |3|$

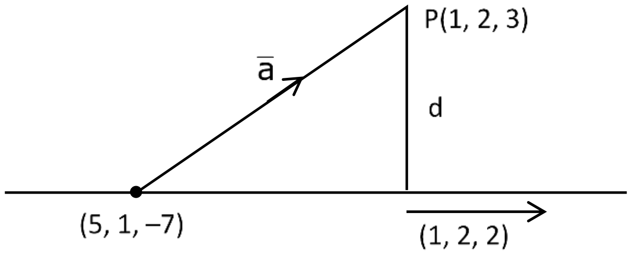
15 – 6λ = 3 or 15 – 6λ = – 3

6λ = 12   6λ = 18

λ = 2 λ = 3

$\lambda_1 = 3$, $\lambda_2 = 2$

$\therefore P(1, 2, 3)$ $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$



$$d = \left|\frac{(4,-1,-10)\times(1,2,2)}{3}\right| = \left|\frac{18\hat{i}-18\hat{j}+9\hat{k}}{3}\right| = 9$$

24. If the solution curve of the differential equation (y-2 $\log_e$ x)dx + (x $\log_e x^2$) dy = 0, x > 1 passes through the points $\left(e, \frac{4}{3}\right)$ and $\left(e^4, \alpha\right)$, then α is equal to ______.

Sol. **(3)**

$$(y - 2\ln x)dx + (2x\ln x)dy = 0$$

$$dy(2x\ln x) = [(2\ln x) - y]dx$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{y}{2x\ln x}$$

$$\frac{dy}{dx} + \frac{y}{2x\ln x} = \frac{1}{x}$$

$$\text{I.F} = e^{\int \frac{1}{2x\ln x}dx}$$

$$= e^{\frac{1}{2}\int \frac{dt}{t}} = e^{\frac{1}{2}\ln(\ln x)}$$

$$\Rightarrow \text{I.F} = (\ln x)^{1/2}$$

$$\therefore y\sqrt{\ln x} = \int \frac{\sqrt{\ln x}}{x}dx \qquad \left(\text{Let}, \ln x = u^2\right)$$

$$= 2\int u^2 du \qquad \frac{1}{x}dx = 2udu$$

$$y\sqrt{\ln x} = \frac{2}{3}(\ln x)^{3/2} + c \leftarrow \left(e, \frac{4}{3}\right)$$

$$\frac{4}{3} = \frac{2}{3} + c \Rightarrow c = \frac{2}{3}$$

$$y\sqrt{\ln x} = \frac{2}{3}(\ln x)^{3/2} + \frac{2}{3} \leftarrow \left(e^4, \alpha\right)$$

$$\alpha \cdot 2 = \frac{2}{3} \times 8 + \frac{2}{3}$$

$$\alpha = 3$$

25. Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}, \vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} - = \vec{a} \times \vec{b}$. If $\vec{a}.\vec{c} = -12, \vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c}.(\hat{i} + \hat{j} + \hat{k})$ is equal to ___.

Sol. **(11)**

$\vec{a} \times \vec{c} = \vec{a} \times 5$

$\Rightarrow \vec{a} \times \left(\vec{c} - \vec{b}\right) = 0$

$\vec{a} \,||^{rl} \left(\vec{c} - \vec{b}\right)$

$\therefore \vec{a} = \lambda\left(\vec{c} - \vec{b}\right)$

$(6,9,12) = \lambda[x - \alpha, y - 11, z + 2]$

$\frac{x - \alpha}{2} = \frac{y - 11}{3} = \frac{z + 2}{4}$

4y – 44 = 3z + 6

4y – 3z = 50

6x + 9y + 12z = – 12

2x + 3y + 4z = – 4 $\quad$ ($\because$ x – 2y + z = 5)

2x – 4y + 2z = 10

$\;\;$ + $\;$ – $\quad$ –

7y + 2z = – 14 …(2)

8y – 6z = 100

21y + 6z = – 42

29y = 58

y = 2, z = – 14

$\therefore$ x – 4 – 14 = 5

x = 23

$\bar{c} = (23, 2, -14)$

$\bar{c} \cdot (1,1,1) = 23 + 2 - 14 = 11$

26. The largest natural number n such that 3^n divides 66! is _______.

Sol. **(31)**

$\left[\frac{66}{3}\right] + \left[\frac{66}{9}\right] + \left[\frac{66}{27}\right]$

22 + 7 + 2 = 31

27. If a_a is the greatest term in the sequence $a_n = \frac{n^3}{n^4 + 147}$, n = 1,2,3,..., then a is equal to _______.

Sol. **(0.158)**

$f(x) = \frac{x^3}{x^4 + 147}$

$f'(x) = \frac{\left(x^4 + 147\right)3x^2 - x^3\left(4x^3\right)}{\left(x^4 + 147\right)^2}$

$= \frac{3x^6 + 147 \times 3x^2 - 4x^6}{+ve} = x^2\left(44 - x^4\right)$

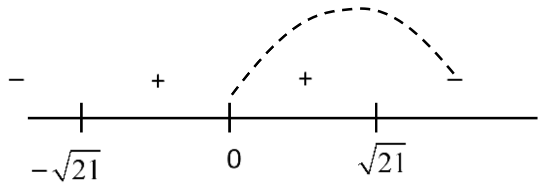
$f'(x) = 0$ at $x^6 = 147 \times 3x^2$

$x^2 = 0, x^4 = 147 \times 3$

$x = 0, x^2 = \pm\sqrt{147 \times 3}$

$x^2 = \pm 21$

$x = \pm\sqrt{21}$



fmax at f(4) or f(5)

$f(4) = \frac{64}{403} \simeq 0.158 \quad f(5) = \frac{125}{772} \simeq 0.161$

$\therefore \; a = 5$

28. Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8 be 9 and 9.25 respectively. If x > y, then 3x -2y is equal to _________.

Sol. **(25)**

$\frac{x + y + 52}{8} = 9 \Rightarrow \; x + y = 20$

For variance

$x - 9, y - 9, 3, 3, 1, -5, -1, -3$

$\bar{x} = 0$

$\therefore \; \frac{(x-9)^2 + (y-9)^2 + 54}{8} - \bar{0}^2 = 9.25$

$(x-9)^2 + (11-x)^2 = 20$

x = 7 or 13 $\therefore$ y = 13, 7

$3x - 2y = 3 \times 13 - 2 \times 7 = 25$

29. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line y = 2x +1 be another circle C_2 : $5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then α + r is equal to _______.

Sol. **(2)**

$x^2 + y^2 - 4x - 2y + 5 - \alpha = 0,$

$C_1(2,1) \; r_1 = \sqrt{\alpha}$

$2x - y + 1 = 0$

Image of (2, 1)

$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{5}$

$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$

$x = 2 - \frac{16}{5} = \frac{-6}{5}, y = 1 + \frac{8}{5} = \frac{13}{5}$

$x^2 + y^2 - 2fx - 2gy + \frac{36}{5} = 0$

$C_2(f, g)$

$r_2 = \sqrt{f^2 + g^2 - \frac{36}{5}}$

$\alpha = f^2 + g^2 - \frac{36}{5}$

$\therefore f = -\frac{6}{5}, g = \frac{13}{5}$

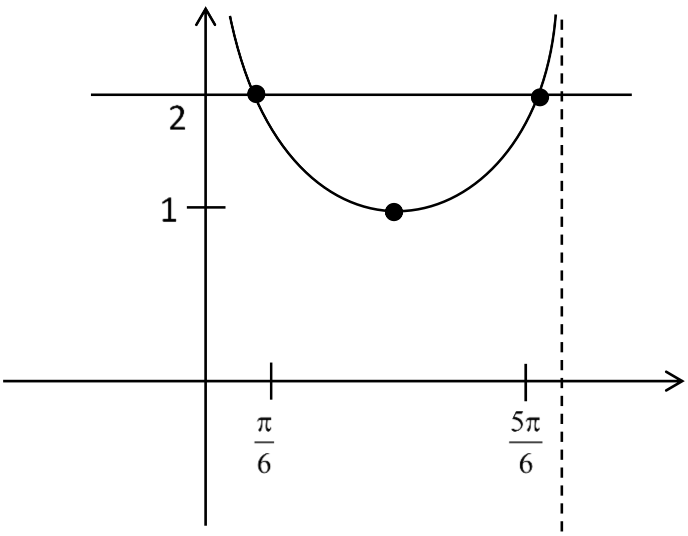
$\alpha = \frac{36}{25} + \frac{169}{25} - \frac{36}{5}$

$= \frac{36 + 169 - 180}{25} \Rightarrow \alpha = 1 \Rightarrow r = 1$

$\therefore \alpha + r = 2$

30. Let [t] denote the greatest integer ≤ t. The $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6}(8[\operatorname{cosec} x] - 5[\cot x])\,dx$ is equal to ______.

Sol. (14)



$8\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\operatorname{cosec} x]dx$

$8\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} dx = \frac{16\pi/3}{16\pi/3}$

$I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\cot x]dx$

$x \to \pi - x$

$I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [-\cot x]dx$

$$2I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\left[\cot x\right] + \left[-\cot x\right]\right) dx$$

$$I = -\frac{1}{2}\int_{\pi/6}^{5\pi/6} dx \Rightarrow -\frac{1}{2}\left(\frac{4\pi}{6}\right)$$

$$= -\pi/3.$$

$$\therefore \frac{2}{\pi}\left[\frac{16\pi}{3} + \frac{5\pi}{3}\right] = \frac{2}{\pi}\left(\frac{21\pi}{3}\right)$$

$$= 14$$

SECTION - A

31. A cylindrical wire of mass (0.4 ± 0.01)g has length (8 ± 0.04) cm and radius (6 ± 0.03) mm. The maximum error in its density will be:

(1) 4% (2) 1% (3) 3.5% (4) 5%

Sol. **(1)**

Cylindrical wire m = (0.4 ± 0.01) g

ℓ = (8 ± 0.04) cm

r = (6 ± 0.03) mm

Density $\rho = \frac{m}{\pi r^2 \ell} \Rightarrow \rho r^2 \ell m^{-1} = \frac{1}{\pi}$ = const.

Differentiating after taking log on both side

$$\frac{d\rho}{\rho} + \frac{2dr}{r} + \frac{d\ell}{\ell} - \frac{dm}{m} = 0$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} - \frac{\Delta\ell}{\ell} - \frac{2\Delta r}{r}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \left[\frac{0.01}{0.4} + \frac{0.04}{8} + 2\left(\frac{0.03}{6}\right)\right]$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = 0.04$$

Percentage error = 0.04 × 100 = 4%

32. The engine of a train moving with speed 10 ms^{-1} towards a platform sounds a whistle at frequency 400 Hz. The frequency heard by a passenger inside the train is : (neglect air speed. Speed of sound in air = 330 ms^{-1})

(1) 400 Hz (2) 388 Hz (3) 200 Hz (4) 412 Hz

Sol. **(1)**

The passenger inside the train is at rest wrt train so frequency heard by passenger inside the train is same as the source frequency i.e., 400 Hz.

33. The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be:

(1) 300 N (2) Zero (3) 100 N (4) 200 N

Sol. **(4)**

Weight on the earth surface = mg

mg = 400 N (given)

Weight at a depth d $w = m\left(\frac{GM(R-d)}{R^3}\right)$

$$W = mg\left(1 - \frac{d}{R}\right)$$

$$d = \frac{R}{2} \Rightarrow w = mg\left(1 - \frac{1}{2}\right) \Rightarrow w = \frac{mg}{2}$$

w = 200 N

34. A TV transmitting antenna is 98 m high and the receiving antenna is at the ground level. If the radius of the earth is 6400 km, the surface area covered by the transmitting antenna is approximately:

(1) 1240 km^2 (2) 1549 km^2 (3) 4868 km^2 (4) 3942 km^2

Sol. **(4)**

Max. distance covered $d = \sqrt{2Rh_T}$

(R = radius of earth, h_T = height of antenna)

Area $A = \pi d^2$

$A = \pi (2Rh_T)$

$A = 2 \times 3.14 \times 6400 \times 98 \times 10^{-3}$

$A \approx 3942\ km^2$

35. Certain galvanometers have a fixed core made of non magnetic metallic material. The function of this metallic material is

(1) To produce large deflecting torque on the coil
(2) To bring the coil to rest quickly
(3) To oscillate the coil in magnetic field for longer period of time
(4) To make the magnetic field radial

Sol. **(2)**

To bring the coil at rest quickly

36. Dimension of $\frac{1}{\mu_0 \in_0}$ should be equal to

(1) T/L (2) T^2 / L^2 (3) L/T (4) L^2/T^2

Sol. **(4)**

Dimension of $\frac{1}{\mu_0\varepsilon_0}$

$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \frac{1}{\mu_0\varepsilon_0} = c^2$

$\left[\frac{1}{\mu_0\varepsilon_0}\right] = [c^2]$

$= \left[\frac{L^2}{T^2}\right]$

37. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is ($g = 10\ m/s^2$)

(1) $2 : \sqrt{3}$ (2) $\sqrt{3} : 2$ (3) 4 : 9 (4) 1 : 1

Sol. **(3)**

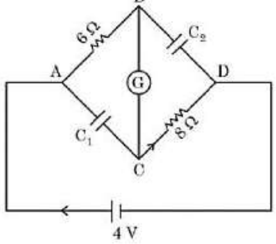
$R = \frac{u^2 \sin 2\theta}{g}$

$\{u_1 = 40\ m/s, \theta_1 = 30°, u_2 = 60\ m/s, \theta_2 = 60°\}$

$\frac{R_1}{R_2} = \left(\frac{u_1}{u_2}\right)^2 \frac{\sin 2\theta_1}{\sin 2\theta_2}$

$\frac{R_1}{R_2} = \left(\frac{40}{60}\right)^2 \times \frac{\sin 60°}{\sin 120°} \Rightarrow \frac{R_1}{R_2} = \frac{4}{9}$

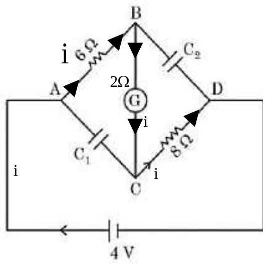
38. In this figure the resistance of the coil of galvanometer G is 2 Ω. The emf of the cell is 4 V. The ratio of potential difference across C_1 and C_2 is:



(1) $\frac{5}{4}$ (2) 1 (3) $\frac{4}{5}$ (4) $\frac{3}{4}$

Sol. **(3)**

At steady state current will not be in the capacitor branch.



$$i = \frac{4}{6+2+8}$$

$$i = \frac{1}{4}A$$

$$\Delta V_{C_1} = i(6+2)$$

$$\Delta V_{C_2} = i(2+8)$$

$$\frac{\Delta V_{C_1}}{\Delta V_{C_2}} = \frac{4}{5}$$

39. A charge particle moving in magnetic field B, has the components of velocity along B as well as perpendicular to B. The path of the charge particle will be

(1) Helical path with the axis along magnetic field B

(2) Straight along the direction of magnetic field B

(3) Helical path with the axis perpendicular to the direction of magnetic field B

(4) Circular path

Sol. **(1)**

Path will be helical with axis along uniform $\vec{B}$ -.

40. Proton (P) and electron (e) will have same de-Broglie wavelength when the ratio of their momentum is (assume, $m_p = 1849\ m_e$):

(1) 1 : 43 (2) 43 : 1 (3) 1 : 1849 (4) 1 : 1

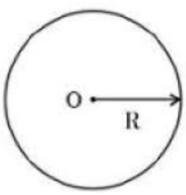
Sol. **(4)**

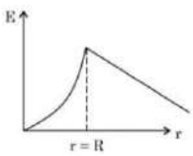
Debroglie wavelength $\lambda = \frac{h}{p}$

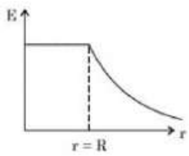
$\lambda_p = \lambda_e$

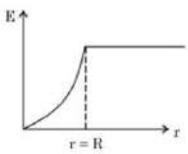
$$\frac{h}{p_p} = \frac{h}{p_e} \Rightarrow \frac{p_p}{p_e} = 1$$

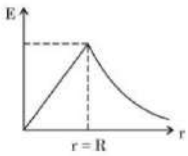
41. Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius R, with distance r from the centre O is represented by:



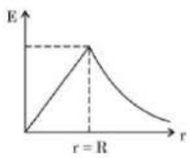
(1) 

(2) 

(3) 

(4) 

Sol. **(4)**



Electric field due to uniformly charged insulating solid sphere

$$E = \begin{cases} \dfrac{kQr}{R^3} & r \le R \\ \dfrac{kQ}{r^2} & r \ge R \end{cases}$$

42. For a nucleus $^{A}_{Z}X$ having mass number A and atomic number Z

A. The surface energy per nucleon $(b_s) = -a_1 A^{2/3}$.

B. The Coulomb contribution to the binding energy $b_c = -a_2 \dfrac{Z(Z-1)}{A^{4/3}}$

C. The volume energy $b_v = a_3A$

D. Decrease in the binding energy is proportional to surface area.

E. While estimating the surface energy, it is assumed that each nucleon interacts with 12 nucleons. (a_1, a_2 and a_3 are constants)

Choose the most appropriate answer from the options given below:

(1) B, C only (2) A, B, C, D only (3) B, C, E only (4) C, D only

Sol. **(4)**

$$E_B = a_vA - a_s A^{2/3} - a_A \frac{(A-2Z)^2}{A^{1/3}} - a_c \frac{Z(Z-1)}{A^{1/3}} + \delta(A,Z)$$

Volume term **Surface term** **Asymmetry term** **Coulomb term** **Pairing term**

Most appropriate is option (4)

43. At any instant the velocity of a particle of mass 500 g is $\left(2t\hat{i} + 3t^2\hat{j}\right)$ ms^{-1}. If the force acting on the particle at t = 1s is $\left(\hat{i} + x\hat{j}\right)$N. Then the value of x will be:

(1) 2 (2) 6 (3) 3 (4) 4

Sol. **(3)**

$\vec{V} = (2t\hat{i} + 3t^2\hat{j})\,m/s$, mass m = 500 gm

Force, $\vec{F} = m\vec{a}$ -

$\vec{F} = \dfrac{1}{2}\left(\dfrac{d\vec{v}}{dt}\right) \Rightarrow \vec{F} = \dfrac{1}{2}\left(2\hat{i} + 6t\hat{j}\right)$

$\vec{F} = \left(\hat{i} + 3t\hat{j}\right)$

At t = 1 s $\Rightarrow$ $\vec{F} = \left(\hat{i} + 3\hat{j}\right)$

x = 3

44. Given below are two statements:

Statement I : If E be the total energy of a satellite moving around the earth, then its potential energy will be $\dfrac{E}{2}$

Statement II : The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both Statement I and Statement II are incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Sol. **(1)**

For satellite K.E. = $\frac{1}{2}$ mv^2 = $\frac{1}{2}$ m $\left(\sqrt{\frac{GM}{r}}\right)^2$

K.E. = $\frac{GMm}{2r}$

Potential energy U = $-\frac{GMm}{r}$

Total energy = K.E + U

E = $-\frac{GMm}{2r}$

U = 2E   St I – incorrect

K.E. = |E| St II - incorrect

45. Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is:

(1) $\frac{5A}{2}$ (2) $\frac{\sqrt{5}A^2}{2}$ (3) $\frac{\sqrt{5}A}{4}$ (4) $\frac{\sqrt{5}A}{2}$

Sol. **(4)**

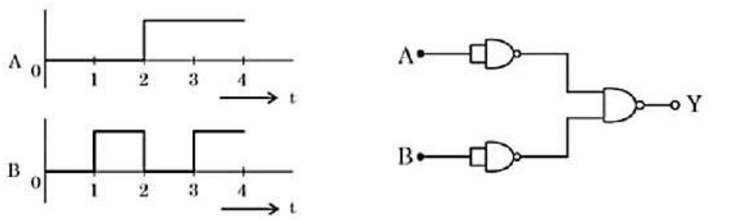
$\left|\vec{F}_1\right| = A\,,\ \left|\vec{F}_2\right| = \frac{A}{2}$ $\theta = \frac{\pi}{2}$

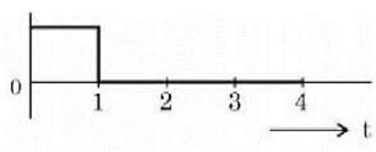
$\left|\vec{F}_{net}\right| = \sqrt{F_1^2 + F_2^2}$

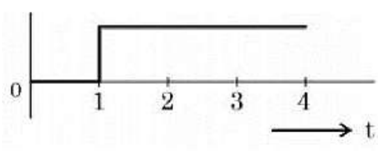
$= \sqrt{A^2 + \left(\frac{A}{2}\right)^2}$

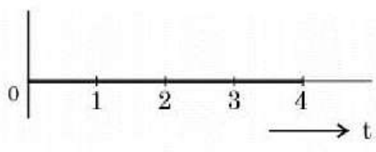
$\left|\vec{F}_{net}\right| = \frac{\sqrt{5}A}{2}$

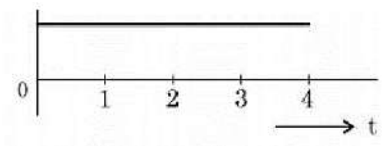
46. For the logic circuit shown, the output waveform at Y is:



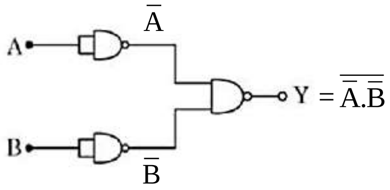
(1)


(2)


(3)


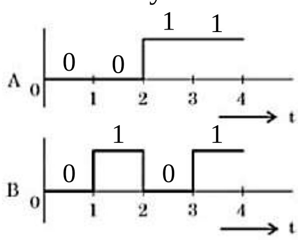
(4)


Sol. **(2)**

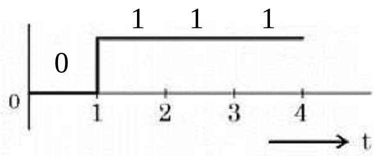


$y = \overline{\overline{A}.\overline{B}} \Rightarrow y = \overline{\overline{A}} + \overline{\overline{B}}$

$y = A + B$



A	B	y = A + B
0	0	0
0	1	1
1	0	1
1	1	1



47. An aluminium rod with Young's modulus $Y = 7.0 \times 10^{10}$ N/m^2 undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is:

(1) 5600 (2) 2800 (3) 11200 (4) 8400

Sol. **(1)**

Aluminium rod Young's modulus

$y = 7.0 \times 10^{10} \dfrac{N}{m^2}$

strain 0.04%

strain $= \dfrac{0.04}{100}$

Energy per unit volume $= \dfrac{1}{2}$ stress × strain

$= \dfrac{1}{2}$ y strain × strain

$= \dfrac{1}{2}$ y (strain)^2

$= \dfrac{1}{2} \times 7 \times 10^{10} \times \left(\dfrac{0.04}{100}\right)^2$

Energy per unit volume $= 5600 \dfrac{J}{m^3}$

48. Given below are two statements:

Statement I : If heat is added to a system, its temperature must increase.

Statement II : If positive work is done by a system in a thermodynamic process, its volume must increase.

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are true (2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false (4) Statement I is false but Statement II is true

Sol. **(4)**

St I False

Ex. in isothermal process temp. is constant but heat can be added.

ST II True

$w = \int PdV$

If volume increases the w = +ve

49. An air bubble of volume 1 cm^3 rises from the bottom of a lake 40 m deep to the surface at a temperature of 12°C. The atmospheric pressure is 1×10^5 Pa, the density of water is 1000 kg/m^3 and g = 10 m/s^2. There is no difference of the temperature of water at the depth of 40 m and on the surface. The volume of air bubble when it reaches the surface will be:

(1) 3 cm^3 (2) 4 cm^3 (3) 2 cm^3 (4) 5 cm^3

Sol. **(4)**

Pressure at surface = $P_{atm} = 1 \times 10^5$ Pa

$v_{surface} = ?$

Pressure at h = 40 m depth

$P = P_{atm} + \rho gh$

$P = 10^5 + 10^3 \times 10 \times 40$

$P = 5 \times 10^5$ Pa

$v = 1\ cm^3$

Temp. is constant

$P_1V_1 = P_2V_2$

$10^5 \times v = 5 \times 10^5 \times 1$

$v = 5\ cm^3$

50. In a reflecting telescope, a secondary mirror is used to:

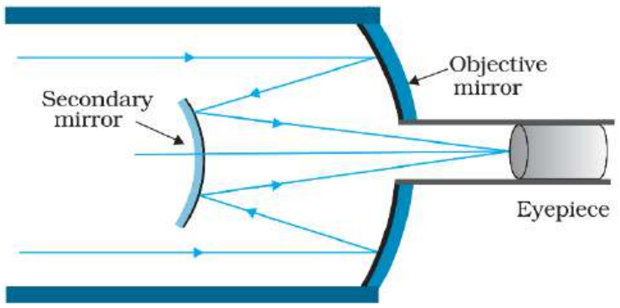
(1) Make chromatic aberration zero

(2) Reduce the problem of mechanical support

(3) Move the eyepiece outside the telescopic tube

(4) Remove spherical aberration

Sol. **(3)**



To move the eye piece outside the telescopic tube

SECTION – B

51. The momentum of a body is increased by 50%. The percentage increase in the kinetic energy of the body is _________ %.

Sol. **(125)**

$$K_i = \frac{P_i^2}{2m}$$

$$K_f = \frac{\left(P_i + \frac{P_i}{2}\right)^2}{2m} \Rightarrow K_f = \frac{9}{4}\frac{P_i^2}{2m}$$

Percentage increase in K.E. = $\frac{K_f - K_i}{K_i} \times 100$

$$= \frac{\frac{9}{4} - 1}{1} \times 100$$

$$= \frac{5}{4} \times 100 = 125\%$$

52. A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into two fragment each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is ___________ MeV.

Sol. (121)

Gain in binding energy = $B.E_f - BE_i$

$= 2(121 \times 8.1) - 242 \times 7.6$

= 121 MeV

53. An electric dipole of dipole moment is 6.0×10^{-6} C m placed in a uniform electric field of 1.5×10^{3} NC^{-1} in such a way that dipole moment is along electric field. The work done in rotating dipole by 180° in this field will be ___________ mJ.

Sol. (18)

$W_{ext} = U_f - U_i \qquad \{U = -\vec{P}.\vec{E}\}$

$= -PE\cos\pi - (-PE\cos 0)$

= 2PE

$= 2 \times 6 \times 10^{-6} \times 1.5 \times 10^{3}$

= 18 mJ

54. An organ pipe 40 cm long is open at both ends. The speed of sound in air is 360 ms^{-1}. The frequency of the second harmonic is ___________ Hz.

Sol. (900)

Open organ pipe ℓ = 40 cm

Speed of sound v = 360 m/s

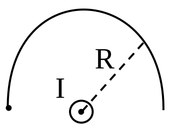
Frequency of second harmonics $f_2 = \frac{2v}{2\ell}$

$f_2 = \frac{v}{\ell} \Rightarrow f_2 = \frac{360}{0.4}$

$f_2 = 900\,Hz$

55. The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is $\frac{1}{x}$ MR^2, where R is the radius and M is the mass of the semicircular ring. The value of x will be ___________.

Sol. (1)



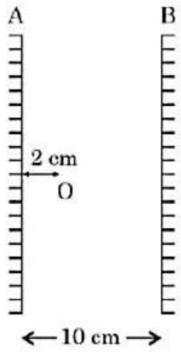
$I = \int dmR^2 \Rightarrow R^2\int dm = MR^2$

$I = MR^2$

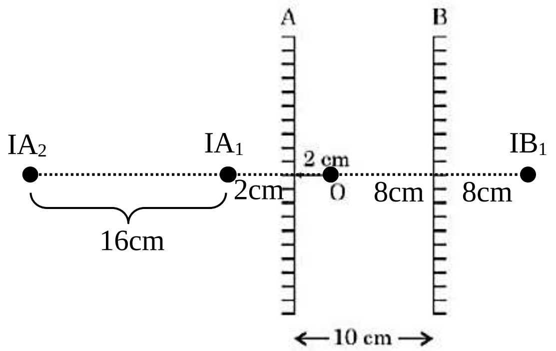
Given I = $\frac{1}{x}MR^2$

x = 1

56. Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is _________ cm.



Sol. **(18)**



d = 2 + 16

d = 18 cm

57. The magnetic intensity at the center of a long current carrying solenoid is found to be 1.6×10^3 Am^{-1}. If the number of turns is 8 per cm, then the current flowing through the solenoid is __________ A.

Sol. **(2)**

$H = 1.6 \times 10^3$ A/m, n = 8 per cm = 800 per m

$H = nI \Rightarrow I = \dfrac{H}{n}$

$I = \dfrac{1.6 \times 10^3}{8 \times 10^2} \Rightarrow I = 2\text{ A}$

58. A current of 2 A through a wire of cross-sectional area 25.0 mm^2. The number of free electrons in a cubic meter are 2.0×10^{28}. The drift velocity of the electrons is ________ $\times 10^{-6}$ ms^{-1}

(given, charge on electron = 1.6×10^{-19} C).

Sol. **(25)**

$I = neAV_d$

$V_d = \dfrac{I}{neA} \Rightarrow V_d = \dfrac{2}{2 \times 10^{28} \times 1.6 \times 10^{-19} \times 25 \times 10^{-6}}$

$V_d = 25$ m/s

59. An oscillating LC circuit consists of a 75 mH inductor and a 1.2 μF capacitor. If the maximum charge to the capacitor is 2.7 μC. The maximum current in the circuit will be __________ mA.

Sol. **(9)**

LC oscillation L = 75 mH

C = 1.2 μF

$U_{max\ L} = U_{max\ C}$

$$\frac{1}{2}\ LI^2_{max} = \frac{1}{2}\ \frac{q^2_{max}}{C}$$

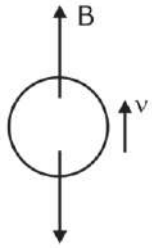
$$I_{max} = \frac{q_{max}}{\sqrt{LC}} \Rightarrow I_{max} = \frac{2.7\times10^{-6}}{\sqrt{75\times10^{-3}\times1.2\times10^{-6}}}$$

$I_{max} = 9 \times 10^{-3}$ A

$I_{max} = 9$ mA

60. An air bubble of diameter 6 mm rises steadily through a solution of density 1750 kg/m^3 at the rate of 0.35 cm/s. TGe co-efficient of viscosity of the solution (neglect density of air) is __________ Pas (given, g = 10 ms^{-2}).

Sol. **(10)**



F_v=6pηrv

For uniform velocity net force = 0

B = 6πηrv

$$\rho\ \frac{4}{3}\pi r^3 g = 6\pi\eta rv$$

$$\eta = \frac{2\rho r^2 g}{9v}$$

$$\eta = \frac{2\times1750\times(3\times10^{-3})^2\times10}{9\times0.35\times10^{-2}}$$

η = 10 Pa-s

SECTION - A

61. The reaction

$\frac{1}{2}H_2(g)+Ag(Cl)(s) \rightleftharpoons H^+(aq)+Cl^-(aq)+Ag(s)$

occurs in which of the given galvanic cell.

(1) $Pt|H_2(g)|HCl(sol^n)|AgNO_3(sol^n)|Ag$
(2) $Pt|H_2(g)|HCl(sol^n)|AgCl(s)|Ag$
(3) $Pt|H_2(g)|KCl(sol^n)|AgCl(s)|Ag$
(4) $Ag|AgCl(s)|KCl(sol^n)|AgNO_3|Ag$

Sol. **2**

Anode $\rightarrow H_2 \rightarrow 2H^+ + 2e^-$

Cathode $\rightarrow AgCl + e^- \rightarrow Ag + Cl^-$

62. Sulphur (S) containing amino acids from the following are:

(a) isoleucine (b) cysteine (c) lysine (d) methionine
(e) glutamic acid

(1) b, c, e (2) a, d (3) a, b, c (4) b, d

Sol. **4**

(a) isoleucine : $CH_3—CH_2—CH(CH_3)—CH(NH_2)—COOH$

(b) cysteine : $HS—CH_2—CH(NH_2)—COOH$

(c) lysine : $H_2N—(CH_2)_4—CH(NH_2)—COOH$

(d) methionine : $CH_3—S—CH_2—CH_2—CH(NH_2)—COOH$

(e) glutamic acid : $HOOC—CH_2—CH_2—CH(NH_2)—COOH$

63. Which of the following complex is octahedral, diamagnetic and the most stable?

(1) $K_3[Co(CN)_6]$ (2) $[Ni(NH_3)_6]Cl_2$ (3) $[Co(H_2O)_6]Cl_2$ (4) $Na_3[CoCl_6]$

Sol. **1**

$K_3[Co(CN)_6]$

$+3 + x - 6 = 0$

$\boxed{x = +3}$

$\Downarrow$

$Co^{+3} \rightarrow 3d^6$

↿⇂	↿	↿	↿	↿

$\because CN^-$ is SFL so pairing occur so

↿⇂	↿⇂	↿⇂		

u-e = 0

$\downarrow$

So diamagnetic

64. Which of the following metals can be extracted through alkali leaching technique?

(1) Cu (2) Au (3) Pb (4) Sn

Sol. **4**

Sn due to Amphoteric nature.

65. The correct order of spin only magnetic moments for the following complex ions is

(1) $[CoF_6]^{3-}$<$[MnBr_4]^{2-}$< $[Fe(CN)_6]^{3-}$<$[Mn(CN)_6]^{3-}$

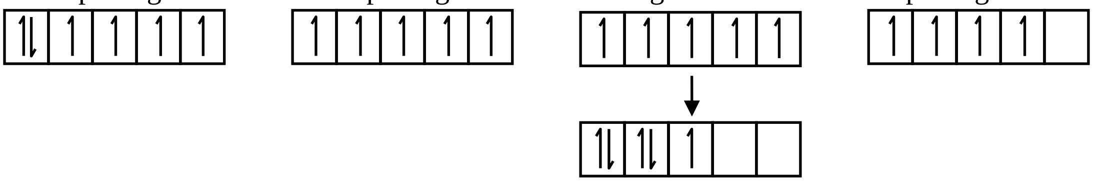
(2) $[Fe(CN)_6]^{3-}$<$[CoF_6]^{3-}$< $[MnBr_4]^{2-}$<$[Mn(CN)_6]^{3-}$

(3) $[MnBr_4]^{2-}$<$[CoF_6]^{3-}$< $[Fe(CN)_6]^{3-}$<$[Mn(CN)_6]^{3-}$

(4) $[Fe(CN)_6]^{3-}$<$[Mn(CN)_6]^{3-}$< $[CoF_6]^{3-}$<$[MnBr_4]^{2-}$

Sol. **4**

$[CoF_6]^{3-}$	$[MnBr_4]^{2-}$	$[Fe(CN)_6]^{3-}$	$[Mn(CN)_6]^{3-}$
↓	↓	↓	↓
CO^{+3}	Mn^{+2}	$3d^5$	Mn^{+3}
F^- WFL	Br^- WFL	CM^- SFL	CN^- is SFL
So no pairing	So no pairing	Pairing occur	So pairing occur



u.e. = 4	u.e. = 5	u.e. = 1	u.e. = 2

$[Fe(CN)_6]^{-3}$ < $[Mn(CN)_6]^{-3}$ < $[CoF_6]^{-3}$ < $[MnBr_4]^{-2}$

66. The water gas on reacting with cobalt as a catalyst forms

(1) Methanoic acid (2) Methanal (3) Ethanol (4) Methanol

Sol. **4**

$$(CO + H_2) + H_2 \xrightarrow[ZnO,\ Cr_2O_3]{700K} CH_3OH$$

↓

Catalyst

67. $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

What is the value of x?

(1) 12 (2) 10 (3) 2 (4) 6

Sol. **2**

n factor of IO_3^- and I^- in the given redox reaction are 5 and 1 respectively. Therefore, IO_3^- will always react in the molar ratio 1 : 5 to get I_2.

$IO_3^- + 6H^+ + 5I^- \rightarrow 3I_2 + 3H_2O$

To get 6 molar I_2, multiple equation by 2

$2IO_3^- + 12H^+ + 10I^- \rightarrow 6I_2 + 6H_2O$

So, x = 10

68. What is the purpose of adding gypsum to cement?

(1) To give a hard mass (2) To speed up the process of setting

(3) To facilitate the hydration of cement (4) To slow down the process of setting

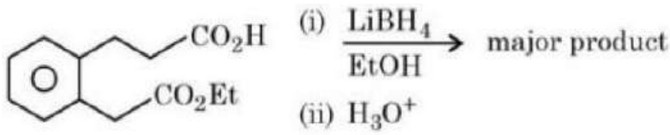
Sol. **4**

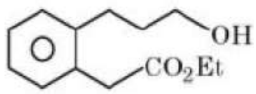
$CaSO_4 \cdot 2H_2O$ Gypsum

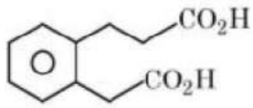
To slow down the process of setting.

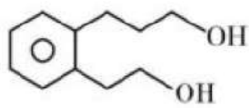
Gypsum is added to control the 'setting of cement'. If not added, the cement will set immediately after miximg of water leaving no time the concrete placing.

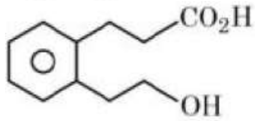
69. The major product formed in the following reaction is:



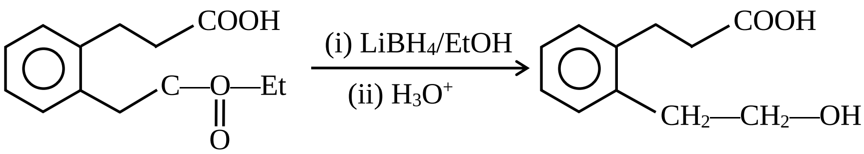
(1) 

(2) 

(3) 

(4) 

Sol. **4**



Note: Lithium borohydride is commonly used for selective reduction of esters and lactones to the corresponding alcohol.

70. Match list I with list II:

List I (species)	**List II (Maximum allowed concentration in ppm in drinking water)**
A. F^-	I. < 50 ppm
B. SO_4^{2-}	II. < 5 ppm
C. NO_3^-	III. < 2 ppm
D. Zn	IV. < 500 ppm

(1) A-III, B-II, C-I, D-IV
(2) A-II, B-I, C-III, D-IV
(3) A-IV, B-III, C-II, D-I
(4) A-I, B-II, C-III, D-IV

Sol. **Bouns**

Data based

	Maximum allowed (ppm)
F^-	< 2 ppm
SO_4^{2-}	< 5 ppm
NO_3^-	< 50 ppm
Zn	< 500 ppm

71. In chromyl chloride, the number of d-electrons present on chromium is same as in (Given at no. of Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26)

(1) Fe (III) (2) V (IV) (3) Ti (III) (4) Mn (VII)

Sol. **4**

$CrO_2Cl_2 \rightarrow$ Chromyl chloride

$\Downarrow$

$Cr^{+6} \rightarrow 4s^0\ 3d^0$

$Mn(vii) \rightarrow Mn^{+7}$ — Same

$\Downarrow$

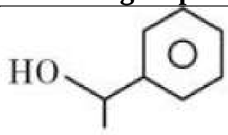
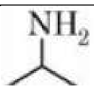
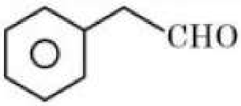
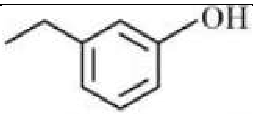
$4s^0\ 3d^0$

72. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Butan-1-ol has higher boiling point than ethoxyethane.
Reason R : Extensive hydrogen bonding leads to stronger association of molecules.
In the light of the above statements, choose the correct answer from the options given below:
(1) Both A and R are true but R is not the correct explanation of A
(2) Both A and R are true and R is the correct explanation of A
(3) A is false but R is true
(4) A is true but R is false

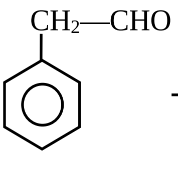
Sol. **2**
At comparable molecular mass, alcohol has higher b.p. than ether due to H-bond, because H-bond leads to stronger associated of molecules.

73. Match List I with List II:

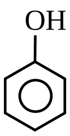
List I (Reagents used)	**List II (Compound with Functional group detected)**
A. Alkaline solution of copper sulphate and sodium cirate	I. 
B. Neutral $FeCl_3$ solution	II. 
C. Alkaline chloroform solution	III. 
D. Potassium iodide and sodium hypochlorite	IV. 

Choose the correct answer from the options given below:
(1) A-III, B-IV, C-II, D-I
(2) A-II, B-IV, C-III, D-I
(3) A-IV, B-I, C-II, D-III
(4) A-III, B-IV, C-I, D-II

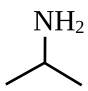
Sol. **1**

(A) 

$\xrightarrow[\text{Sulphate and sodium citrate}]{\text{Alkaline solution of copper}}$ Red PPt

(B) Neutral $FeCl_3$ 

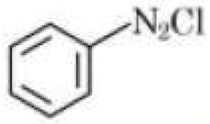
$\longrightarrow$ Voilet colour

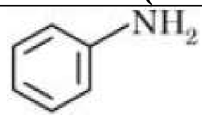
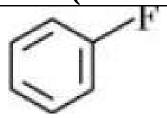
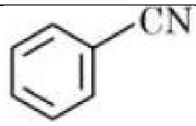
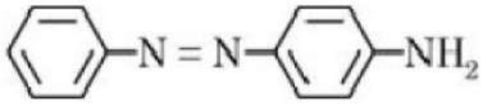
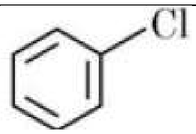
(C) Alkaline chloroform solution 

$\longrightarrow$ Carbyl amine reaction

(D) KI + NaOCl $\xrightarrow{CH_3-CH(OH)-Ph}$ Haloform reaction

74. Match List I with List II:

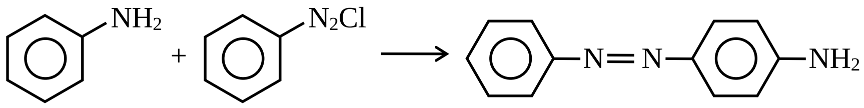
 is reacted with reagents in List I to form products in List II.

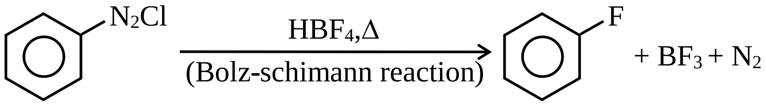
List I (Reagent)	List II (Product)
A. 	I. 
B. HBF_4, Δ	II. 
C. Cu, HCl	III. 
D. CuCN/KCN	IV. 

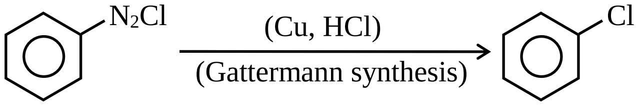
Choose the correct answer from the options given below:

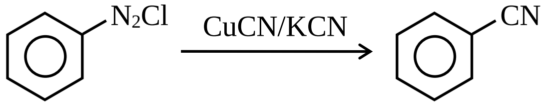
(1) A-I, B-III, C-IV, D-II
(2) A-III, B-I, C-II, D-IV
(3) A-III, B-I, C-IV, D-II
(4) A-IV, B-III, C-II, D-I

Sol. 3

(A) 

(B) 

(C) 

(D) 

75. Match List I with List II:

List I	List II
A. Saccharin	I. High potency sweetener
B. Aspartame	II. First artificial sweetening agent
C. Alitame	III. Stable at cooking temperature
D. Sucralose	IV. Unstable at cooking temperature

Choose the **correct** answer from the options given below:

(1) A-II, B-III, C-IV, D-I
(2) A-II, B-IV, C-I, D-III
(3) A-IV, B-III, C-I, D-II
(4) A-II, B-IV, C-III, D-I

Sol. 2

(A) Saccharin → First artificial sweetening agent
(B) Aspartame → Unstable at cooking temperature used in soft drink and cold drink.
(C) Alitame → High potency sweetener (2000 more sweeter than cane sugar)
(D) Sucralose → Stable at coocking temperature. Also it does not provide calories.

76. The correct order of electronegativity for given elements is:

(1) P > Br > C > At (2) C > P > At > Br (3) Br > P > At > C (4) Br > C > At > P

Sol. **4**

C (2.5)

P (2.1) $\Rightarrow$ Br > C > At > P

Br (2.8)

At (2.2)

77. Given below are two statements :

Statement I : Lithium and Magnesium do not form superoxide

Statement II : The ionic radius of Li^+ is larger than ionic radius of Mg^{2+}

In the light of the above statements, choose the **most appropriate** answer from the options given **below**:

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both statement I and Statement II are correct

(4) Both statement I and Statement II are incorrect

Sol. **3 (Fact-based)**

Due to small in size Li and Mg do not from superoxide.

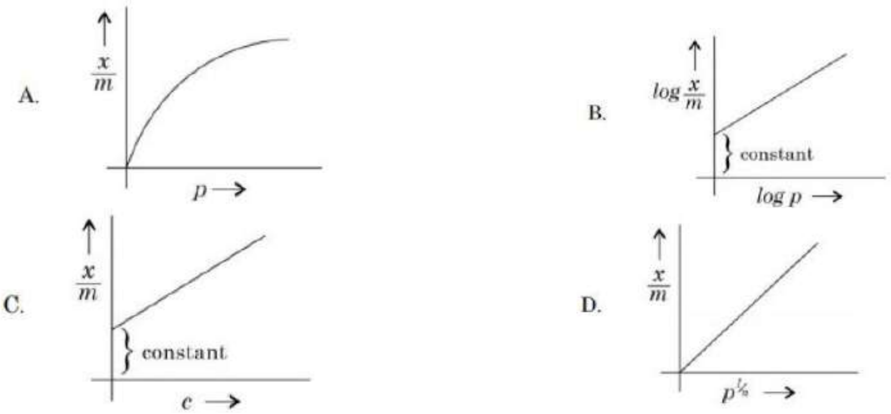
$Li^+ \geq Mg^{+2}$ - radius

$2e^-$ $10e^-$

$\downarrow$

Due to diagonal relationship.

78. Which of the following represent the Freundlich adsorption isotherms?

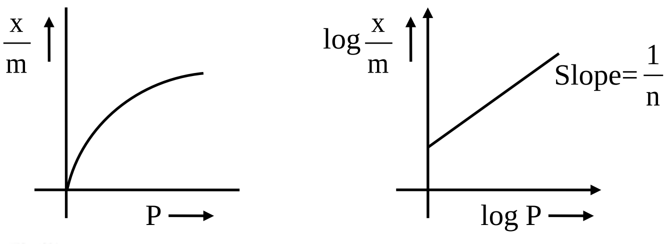


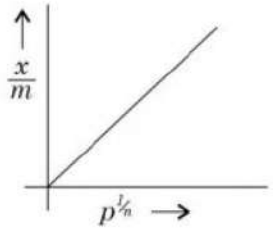
Choose the correct answer from the options given below:

(1) A, C, D only (2) A, B only (3) A, B, D only (4) B, C, D only

Sol. **3**

$\frac{x}{m} = Kp^{1/n}$ $\log\frac{x}{m} = \log K + \frac{1}{n}\log P$





79. Which halogen is known to cause the reaction given below:

$2Cu^{2+} + 4X^{-} \rightarrow Cu_2X_2(s) + X_2$

(1) All halogens (2) Only chlorine (3) Only Bromine (4) Only Iodine

Sol. **4**

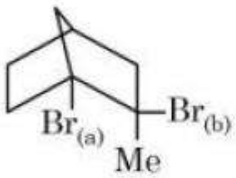
(Only iodine)

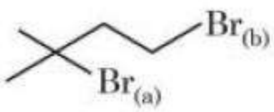
$2Cu^{2+} + 4I^{-} \rightarrow Cu_2I_2 + I_2$

80. Choose the halogen which is most reactive towards S_N1 reaction in the given compounds (A, B, C, & D)

A. 

B. 

C. 

D. 

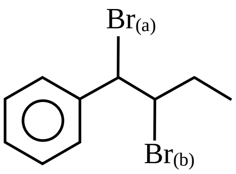
(1) A-$Br_{(a)}$; B-$I_{(a)}$; C-$Br_{(b)}$; D-$Br_{(a)}$

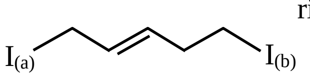
(2) A-$Br_{(b)}$; B-$I_{(a)}$; C-$Br_{(a)}$; D-$Br_{(a)}$

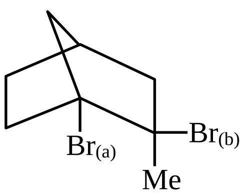
(3) A-$Br_{(b)}$; B-$I_{(b)}$; C-$Br_{(b)}$; D-$Br_{(b)}$

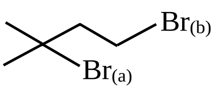
(4) A-$Br_{(a)}$; B-$I_{(a)}$; C-$Br_{(a)}$; D-$Br_{(a)}$

Sol. **1**

(A)  → Because formed intermediate carbocation formed by $Br_{(a)}$ get stabilised by conjugation with phenyl ring

(B)  → Because the intermediate carbocation formed by $I_{(a)}$ become more stable by conjugation

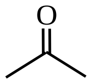
(C)  → Becouse, we con't remove $Br_{(a)}$ from bridge head carbon (Bredt's rule)

(D)  → Becouse, formed intermediate by $Br_{(a)}$, 3° carbocation is more stable (stability of carbocation 3° > 2° > 1°)

SECTION - B

81. Molar mass of the hydrocarbon (X) which on ozonolysis consumes one mole of O_3 per mole of (X) and gives one mole each of ethanol and propanone is ___________g mol^{-1} (Molar mass of C : 12 g mol^{-1}, H : 1 $gmol^{-1}$)

Sol. **70**

Reactant $\xrightarrow{O_3}$  + CH_3CHO

$$CH_3-\overset{\overset{\large CH_3}{|}}{C}=CH-CH_3$$

(C_5H_{10})

Molecular Mass = 70

82. XeF_4 reacts with SbF_5 to form

$[XeFm]^{n+}$ $[SbF_y]^{z-}$

m+n+y+z =_________

Sol. **11**

$XeF_4 + SbF_5 \rightarrow [XeF_3]^+ (SbF_6)^-$

m + n + x + y = 3 + 1 + 6 + 1 = 11

Xenon fluoride act as F^- donor and F^- acceptor.

83. The number of following statements which is/are incorrect is_________

(1) Line emission spectra are used to study the electronic structure

(2) The emission spectra of atoms in the gas phase show a continuous spread of wavelength from red to violet

(3) An absorption spectrum is like the photographic negative of an emission spectrum

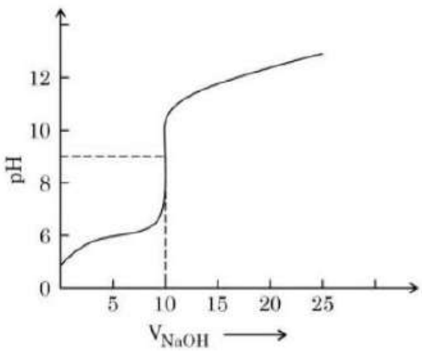
(4) The element helium was discovered in the sun by spectroscopic method

Sol. **1**

Fact

84. The titration curve of weak acid vs. strong base with phenolphthalein as indictor) is shown below. The $K_{phenolphthalein} = 4\times10^{-10}$

Given : log 2 = 0.3



The number of following statements/s which is/are correct about phenolphthalein is__________

(1) It can be used as an indicator for the titration of weak acid with weak base.

(2) It begins to change colour at pH = 8.4

(3) It is a weak organic base

(4) It is colourless in acidic medium

Sol. **2**

(B) $pk_n = -\log(4\times10^{-10}) = 9.4$

Indicator range

$\Rightarrow$ $pk_{In} \pm 1$

i.e. 8.4 to 10.4

(D) In acidic medium, phenolphthalein is in unionized form and is colourless.

85. When a 60 W electric heater is immersed in a gas for 100s in a constant volume container with adiabatic walls, the temperature of the gas rises by 5ºC. The heat capacity of the given gas is_______J K^{-1} (Nearest integer)

Sol. **1200**

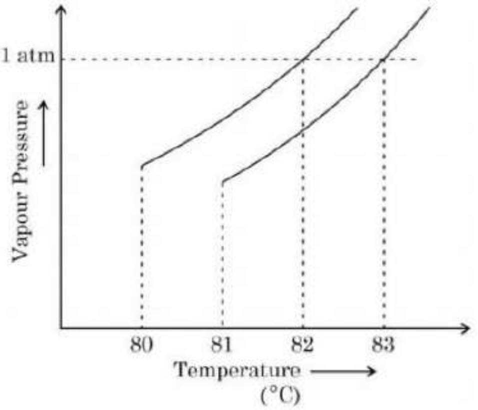
Adiabatic wall {no heat exchange between system and surrounding}

$C_v \times \Delta T = P \times t/sec$

$C_v \times 5 = 60 \times 100$

$C_v = 1200$

86. The vapour pressure vs. temperature curve for a solution solvent system is shown below:



The boiling point of the solvent is______ºC

Sol. **82**

Boiling point of solvent is 82ºC

Boiling point of solvent is 83ºC

87. 0.5 g of an organic compound (X) with 60% carbon will produce______$\times 10^{-1}$ g of CO_2 on complete combustion.

Sol. **11**

Moles of carbon = $\frac{0.5 \times 0.6}{12}$

Moles of CO_2 = $\frac{0.5 \times 0.6}{12}$

Mass of CO_2 = $\frac{0.5 \times 0.6}{12} \times 44 = 11 \times 10^{-1}$ gram

88. The number of following factors which affect the percent covalent character of the ionic bond is_____

(1) Polarising power of cation (2) Extent of distortion of anion

(3) Polarisability of the anion (4) Polarising power of anion

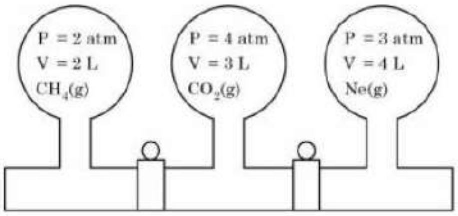
Sol. **3**

Percent covalent character of the ionic bond

(1) Polarising power of cation (2) Extent of distortion of anion

(3) Polarisability of the anion

89.



Three bulbs are filled with CH_4, CO_2 and Ne as shown the picture. The bulbs are connected through pipes of zero volume. When the stopcocks are opened and the temperature is kept constant throughout, the pressure of the system is found to be________atm. (Nearest integer)

Sol. **3**

$P_f V_f = P_1 V_1 + P_2 V_2 + P_3 V_3$

$P_f \times 9 = 2 \times 2 + 4 \times 3 + 3 \times 4$

$$P_f = \frac{28}{9} = 3.11 \simeq 3$$

90. The number of given statements/s which is/are correct is______

(1) The stronger the temperature dependence of the rate constant, the higher is the activation energy.

(2) If a reaction has zero activation energy, its rate is independent of temperature.

(3) The stronger the temperature dependence of the rate constant, the smaller is the activation energy

(4) If there is no correlation between the temperature and the rate constant then it means that the reaction has negative activation energy.

Sol. **2**

Clearly, if $E_a = 0$, K is temperature independent

if $E_a > 0$, K increase with increase in temperature

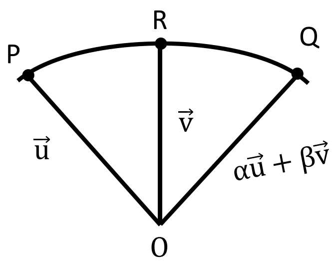
if $E_a < 0$, K decrease with increase in temperature

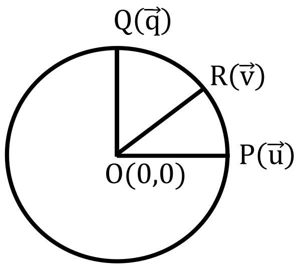
SECTION-A

1. An arc PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. If $\overrightarrow{OP} = \vec{u}, \overrightarrow{OR} = \vec{v}$ and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation :

(1) $3x^2 - 2x - 1 = 0$ (2) $3x^2 + 2x - 1 = 0$ (3) $x^2 - x - 2 = 0$ (4) $x^2 + x - 2 = 0$

Sol. **(3)**





Let $\overrightarrow{OP} = \vec{u} = \hat{i}$

$\overrightarrow{OQ} = \vec{q} = \hat{j}$

$\because$ R is the mid point of $\overrightarrow{PQ}$

Then $\overrightarrow{OR} = \vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

Now

$\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$

$$\hat{j} = \alpha\hat{i} + \beta\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\beta = \sqrt{2}, \alpha + \frac{\beta}{\sqrt{2}} = 0 \Rightarrow \alpha = -1$$

Now equation

$$x^2 - \left(\alpha + \beta^2\right)x + \alpha\beta^2 = 0$$

$$x^2 - \left(-1 + 2\right)x + (-1)(2) = 0$$

$$\boxed{x^2 - x - 2 = 0}$$

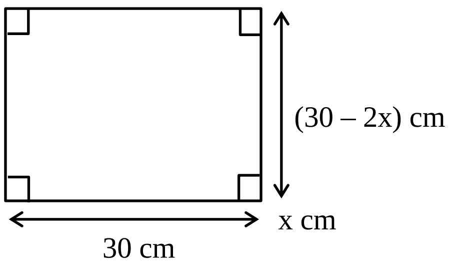
2. A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm^2) is equal to :

(1) 800 (2) 1025 (3) 900 (4) 675

Sol. **(1)**

Let the side of the square to be cut off be x cm.

Then, the length and breadth of the box will be (30 – 2x) cm each and the height of the box is x cm therefore,



The volume V(x) of the box is given by

$V(x) = x(30 - 2x)^2$

$\frac{dv}{dx} = (30 - 2x)^2 + 2x \times (30 - 2x)(-2)$

$0 = (30 - 2x)^2 - 4x(30 - 2x)$

$0 = (30 - 2x)[(30 - 2x) - 4x]$

$0 = (30 - 2x)(30 - 6x)$

$x = 15, 5$

$x \neq 15$ (Not possible)

$\{\therefore V = 0\}$

Surface area without top of the box = $\ell b + 2(bh + h\ell)$

$= (30 - 2x)(30 - 2x) + 2[(30 - 2x)x + (30 - 2x)x]$

$= [(30 - 2x)^2 + 4\{(30 - 2x)x\}$

$= [(30 - 10)^2 + 4(5)(30 - 10)]$

$= 400 + 400$

$= 800 \text{ cm}^2$

3. Let O be the origin and the position vector of the point P be $-\hat{i} - 2\hat{j} + 3\hat{k}$. If the position vectors of the A, B and C are $-2\hat{i} + \hat{j} - 3\hat{k}, 2\hat{i} + 4\hat{j} - 2\hat{k}$ and $-4\hat{i} + 2\hat{j} - \hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is :

(1) $\frac{10}{3}$ (2) $\frac{8}{3}$ (3) $\frac{7}{3}$ (4) 3

Sol. **(4)**

Position vector of the point $P(-1, -2, 3)$, $A(-2, 1, -3)$ $B(2, 4, -2)$, and $C(-4, 2, -1)$

Then $\overrightarrow{OP}.\frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|(\overrightarrow{AB} \times \overrightarrow{AC})|}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 1 \\ -2 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(5) - \hat{j}(8 + 2) + \hat{k}(4 + 6)$

$= 5\hat{i} - 10\hat{j} + 10\hat{k}$

Now

$$\overrightarrow{OP}.\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{\left|(\overrightarrow{AB}\times\overrightarrow{AC})\right|}=(-\hat{i}-2\hat{j}+3\hat{k}).\frac{(5\hat{i}-10\hat{j}+10\hat{k})}{\sqrt{(5)^2+(-10)^2+(10)^2}}$$

$$=\frac{-5+20+30}{\sqrt{25+100+100}}$$

$$=\frac{45}{\sqrt{225}}=\frac{45}{15}=3$$

4. If A is a 3 × 3 matrix and |A| = 2, then $\left|3\text{adj}\left(|3A|A^2\right)\right|$ is equal to :

(1) $3^{12}.\ 6^{10}$ (2) $3^{11}.6^{10}$ (3) $3^{12}.6^{11}$ (4) $3^{10}.6^{11}$

Sol. **(2)**

Given |A|= **2**

Now, $|3\text{adj}\ (\ |\ 3A|\ A^2)\ |$

$|3A| = 3^3.|A|$

$= 3^3.(2)$

$\text{Adj.}\ (|3A|\ A^2) = \text{adj}\ \{(3^3.2)\ A^2\}$

$= (2.3^3)^2\ (\text{adj } A)^2$

$= 2^2.3^6\ .(\text{adj } A)^2$

$|3\ \text{adj}\ (|\ 3A\ |\ A^2)\ | = |2^2.3.3^6\ (\text{adj } A)^2|$

$= (2^2.3^7)^3\ |\ \text{adj } A|^2$

$= 2^6.3^{21}\ (|A|^2)^2$

$= 2^6.3^{21}(2^2)^2$

$= 2^{10}.3^{21}$

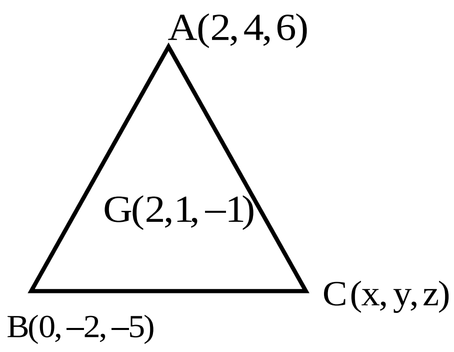
$= 2^{10}.3^{10}.3^{11}$

$|3\ \text{adj}\ (|\ 3A\ |\ A^2)| = 6^{10}.3^{11}$

5. Let two vertices of a triangle ABC be (2, 4, 6) and (0, –2, –5), and its centroid be (2, 1, –1). If the image of the third vertex in the plane x + 2y + 4z = 11 is (α, β, γ), then αβ + βγ + γα is equal to :

(1) 76 (2) 74 (3) 70 (4) 72

Sol. **(2)**



Given Two vertices of Triangle $A(2,4,6)$ and $B(0,-2,-5)$ and if centroid $G(2,1,-1)$

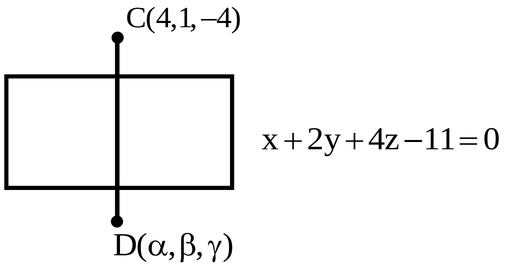
Let Third vertices be (x,y,z)

Now $\frac{2+0+x}{3}=2,\frac{4-2+y}{3}=1,\frac{6-5+z}{3}=-1$

$x=4,\ \ y=1,\ \ \ z=-1$

Third vertices $C(4,1,-4)$

Now, Image of vertices C(4,1,–4) in the given plane is D(α,β,γ)



Now

$$\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=-2\frac{(4+2-16-11)}{1+4+16}$$

$$\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=\frac{42}{21}\Rightarrow 2$$

$$\alpha=6,\ \beta=5,\ \gamma=4$$

Then $\alpha\beta+\beta\gamma+\gamma\alpha$

$$=(6\times5)+(5\times4)+(4\times6)$$

$$=30+20+24$$

$$=74$$

6. The negation of the statement :

$(p \vee q) \wedge (q \vee (\sim r))$ is

(1) $((\sim p) \vee r)) \wedge (\sim q)$ (2) $((\sim p) \vee (\sim q)) \wedge (\sim r)$

(3) $((\sim p) \vee (\sim q)) \vee (\sim r)$ (4) $(p \vee r) \wedge (\sim q)$

Sol. **(1)**

$$(p\vee q)\wedge(q\vee(\sim r))$$

$$\sim\left[(p\vee q)\wedge(q\vee(\sim r))\right]$$

$$=\sim(p\vee q)\wedge(\sim q\wedge r)$$

$$=(\sim p\wedge\sim q)\vee(\sim q\wedge r)$$

$$=(\sim p\vee r)\wedge(\sim q)$$

7. The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is :

(1) 8 (2) 7 (3) 6 (4) 9

Sol. **(4)**

$$\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}\ \text{and}\ \frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$$

$$\text{Shortest distance } (d) = \frac{\left\| \begin{matrix} a_2 - a_1 & b_2 - b_1 & c_2 - c_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{matrix} \right\|}{\left\| \begin{matrix} \hat{i} & \hat{j} & k \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{matrix} \right\|}$$

$$= \frac{\left\| \begin{matrix} 4+2 & 1-0 & -3-5 \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{matrix} \right\|}{\left\| \begin{matrix} \hat{i} & \hat{j} & k \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{matrix} \right\|}$$

$$= \frac{\left| \begin{matrix} 6 & 1 & -8 \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{matrix} \right|}{\left| \hat{i}(-4) - \hat{j}(-2) + k(2+2) \right|}$$

$$= \frac{|-54|}{\left| -4\hat{i} + 2\hat{j} + 4k \right|}$$

$$= \frac{54}{\sqrt{16+4+16}}$$

$$= \frac{54}{6}$$

$$= 9$$

8. If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to :

(1) 22 (2) 44 (3) 11 (4) 33

Sol. **(1)**

$$\left(ax - \frac{1}{bx^2}\right)^{13}$$

We have,

$T_{r+1} = {}^nC_r\,(p)^{n-r}\,(q)^r$

$T_{r+1} = {}^{13}C_r\,(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$

$= {}^{13}C_r\,(a)^{13-r}\left(-\frac{1}{b}\right)^r (x)^{13-r}.(x)^{-2r}$

$= {}^{13}C_r\,(a)^{13-r}\left(-\frac{1}{b}\right)^r (x)^{13-3r}$ …(1)

Coefficient of x^7
$\Rightarrow 13 - 3r = 7$
$r = 2$
r in equation (1)

$T_3 = {}^{13}C_2 (a)^{13-2} \left(-\frac{1}{b}\right)^2 (x)^{13-6}$

$= {}^{13}C_2 (a)^{11} \left(\frac{1}{b}\right)^2 (x)^7$

Coefficient of x^7 is ${}^{13}C_2 \frac{(a)^{11}}{b^2}$

Now, $\left(ax + \frac{1}{bx^2}\right)^{13}$

$T_{r+1} = {}^{13}C_r (ax)^{13-r} \left(\frac{1}{bx^2}\right)^r$

$= {}^{13}C_r (a)^{13-r} \left(\frac{1}{b}\right)^r (x)^{13-r}(x)^{-2r}$

$= {}^{13}C_r (a)^{13-r} \left(\frac{1}{b}\right)^r (x)^{13-3r}$ ...(2)

Coefficient of x^{-5}
$\Rightarrow 13 - 3r = -5$
$r = 6$
r in equation

$T_7 = {}^{13}C_6 (a)^{13-6} \left(\frac{1}{b}\right)^6 (x)^{13-18}$

$T_7 = {}^{13}C_6 (a)^7 \left(\frac{1}{b}\right)^6 (x)^{-5}$

Coefficient of x^{-5} is ${}^{13}C_6 (a)^7 \left(\frac{1}{b}\right)^6$

ATQ
Coefficient of x^7 = coefficient of x^{-5}
$T_3 = T_7$

${}^{13}C_2 \left(\frac{a^{11}}{b^2}\right) = {}^{13}C_6 (a)^7 \left(\frac{1}{b}\right)^6$

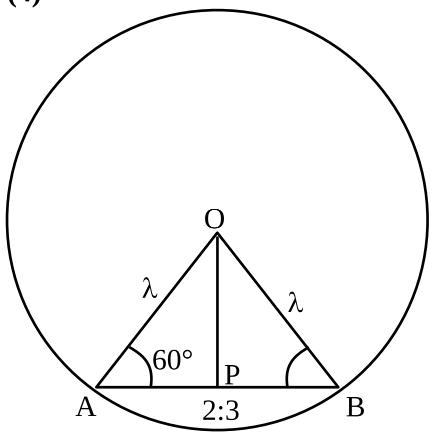
$a^4.b^4 = \frac{{}^{13}C_6}{{}^{13}C_2}$

$= \frac{13\times12\times11\times10\times9\times8\times1}{13\times12\times6\times5\times4\times3} = 22$

9. A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius :

(1) $\frac{2}{3}\lambda$ (2) $\frac{\sqrt{19}}{7}\lambda$ (3) $\frac{3}{5}\lambda$ (4) $\frac{\sqrt{19}}{5}\lambda$

Sol. **(4)**



Since OAB form equilateral Δ

$\therefore \angle OAP = 60^o$

$AP = \frac{2\lambda}{5}$

$\cos 60° = \frac{OA^2 + AP^2 - OP^2}{2OA.AP}$

$\Rightarrow \frac{1}{2} = \frac{\lambda^2 + \frac{4\lambda^2}{25} - OP^2}{2\lambda\left(\frac{2\lambda}{5}\right)}$

$\Rightarrow \frac{2\lambda^2}{5} = \lambda^2 + \frac{4\lambda^2}{25} - OP^2$

$\Rightarrow OP^2 = \frac{19\lambda^2}{25}$

$\Rightarrow OP = \frac{\sqrt{19}}{5}\lambda$

Therefore, locus of point P is $\frac{\sqrt{19}}{5}\lambda$

10. For the system of linear equations

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta$,

which of the following is NOT correct ?

(1) The system in inconsistent for $\alpha = -5$ and $\beta = 8$

(2) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

(3) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$

(4) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

Sol. **(2)**

Given

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta$

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \alpha \end{vmatrix} = 7\alpha + 35$$

$\Delta = 7(\alpha + 5)$

For unique solution $\Delta \neq 0$

$\alpha \neq -5$

For inconsistent & Infinite solution

$\Delta = 0$

$\alpha + 5 = 0 \Rightarrow \alpha = -5$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & 3 \\ 7 & 2 & -1 \\ \beta & 5 & -5 \end{vmatrix} = -5(\beta - 9)$$

$$\Delta_2 = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 7 & -1 \\ 4 & \beta & -5 \end{vmatrix} = 11(\beta - 9)$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 2 & 7 \\ 4 & 5 & \beta \end{vmatrix}$$

$\Delta_3 = 7(\beta - 9)$

For Inconsistent system : -

At least one Δ_1, Δ_2 & Δ_3 is not zero $\alpha = -5$, $\beta = 8$ option (A) True

Infinite solution:

$\Delta_1 = \Delta_2 = \Delta_3 = 0$

From here $\beta - 9 = 0 \Rightarrow \beta = 9$

$\alpha = -5$ & option (D) True

$\beta = 9$

Unique solution

$\alpha \neq -5$, $\beta = 8 \rightarrow$ option (C) True

Option (B) False

For Infinitely many solution α must be -5.

11. Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three is 33033, then the sum of these terms is equal to :

(1) 210 (2) 220 (3) 231 (4) 241

Sol. **(3)**

Let a, ar , ar^2 be three terms of GP

Given : $a^2 + (ar)^2 + (ar^2)^2 = 33033$

$a^2(1 + r^2 + r^4) = 11^2.3.7.13$

$\Rightarrow a = 11$ and $1 + r^2 + r^4 = 3.7.13$

$\Rightarrow r^2(1 + r^2) = 273 - 1$

$\Rightarrow r^2(r^2 + 1) = 272 = 16 \times 17$

$\Rightarrow r^2 = 16$

$\therefore r = 4$ $[\because r > 0]$

Sum of three terms $= a + ar + ar^2 = a(1 + r + r^2)$

$= 11(1 + 4 + 16)$

$= 11 \times 21 = 231$

12. Let P be the point of intersection of the line $\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}$ and the plane x + y + z = 2. If the distance of the point P from the plane 3x – 4y + 12z = 32 is q, then q and 2q are the roots of the equation :

(1) $x^2 + 18x - 72 = 0$ (2) $x^2 + 18x + 72 = 0$ (3) $x^2 - 18x - 72 = 0$ (4) $x^2 - 18x + 72 = 0$

Sol. **(4)**

$\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2} = \lambda$

$x = 3\lambda - 3,\ y = \lambda - 2,\ z = 1 - 2\lambda$

$P(3\lambda - 3, \lambda - 2, 1 - 2\lambda)$ will satisfy the equation of plane x + y + z = 2.

$3\lambda - 3 + \lambda - 2 + 1 - 2\lambda = 2$

$2\lambda - 4 = 2$

$\lambda = 3$

P(6, 1, – 5)

Perpendicular distance of P from plane 3x – 4y + 12 z – 32 = 0 is

$q = \left|\frac{3(6)-4(1)+12(-5)-32}{\sqrt{9+16+144}}\right|$

q = 6.

2q = 12

Sum of roots = 6 + 12 = 18

Product of roots = 6(12) = 72

$\therefore$ Quadratic equation having q and 2q as roots is $x^2 - 18x + 72$.

13. Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^x tf(t)\,dt$, $f(1) = \frac{2}{3}$. Then 18 f (3) is equal to :

(1) 180 (2) 150 (3) 210 (4) 160

Sol. **(4)**

$x^2 f(x) - x = 4\int_0^x tf(t)dt$

Differentiate w.r.t. x

$x^2 f'(x) + 2x\, f(x) - 1 = 4xf(x)$

Let y = f(x)

$\Rightarrow x^2 \frac{dy}{dx} - 2xy - 1 = 0$

$\frac{dy}{dx} - \frac{2}{x} y = \frac{1}{x^2}$

I.F. $= e^{\int \frac{-2}{x}dx} = \frac{1}{x^2}$

Its solution is

$\frac{y}{x^2} = \int \frac{1}{x^4}\, dx + C$

$\frac{y}{x^2} = \frac{-1}{3x^3} + C$

$\because f(1) = \frac{2}{3} \Rightarrow y(1) = \frac{2}{3}$

$\Rightarrow \frac{2}{3} = -\frac{1}{3} + C$

$\Rightarrow C = 1$

$\because y = -\frac{1}{3x} + x^2$

$f(x) = -\frac{1}{3x} + x^2$

$f(3) = -\frac{1}{9} + 9 = \frac{80}{9} \Rightarrow 18f(3) = 160$

14. Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then 4m –3n equal to :

(1) 12 (2) 8 (3) 10 (4) 6

Sol. **(2)**

$2^N < N!$ is satisfied for $N \geq 4$

Required probability $P(N \geq 4) = 1 - P(N < 4)$

N = 1 (Not possible)

N = 2 (1, 1)

$\Rightarrow P(N = 2) = \frac{1}{36}$

N = 3 (1, 2), (2, 1)

$\Rightarrow P(N = 3) = \frac{2}{36}$

$P(N < 4) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$

$\therefore P(N \geq 4) = 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12} = \frac{m}{n}$

$\Rightarrow m = 11, n = 12$

$\therefore 4m - 3n = 4(11) - 3(12) = 8$

15. If $I(x) = \int e^{\sin^2 x} (\cos x \sin 2x - \sin x)\, dx$ and I (0) = 1, then $I\left(\frac{\pi}{3}\right)$ is equal to :

(1) $e^{\frac{3}{4}}$ (2) $-e^{\frac{3}{4}}$ (3) $\frac{1}{2}e^{\frac{3}{4}}$ (4) $-\frac{1}{2}e^{\frac{3}{4}}$

Sol. **(3)**

$I = \int \underbrace{e^{\sin^2 x} \sin 2x}_{II} \underbrace{\cos x dx}_{I} - \int e^{\sin^2 x} \sin x dx$

$= \cos x \int e^{\sin^2 x} \sin 2x dx - \int \left((-\sin x) \int e^{\sin^2 x} \sin 2x dx \right) dx - \int e^{\sin^2 x} \sin x dx$

$\sin^2 x = t$

$\sin 2x\, dx = dt$

$= \cos x \int e^t dt + \int (\sin x \int e^t dt) dx - \int e^{\sin^2 x} \sin x dx$

$= e^{\sin^2 x} \cos x + \int e^{\sin^2 x} \sin x dx - \int e^{\sin^2 x} \sin x dx$

$I = e^{\sin^2 x}\cos x + C$

I (0) = 1

$\Rightarrow$ 1 = 1 + C

$\Rightarrow$ C = 0

$\therefore\ I = e^{\sin^2 x}\cos x$

$I\left(\frac{\pi}{3}\right) = e^{\sin^2\frac{\pi}{3}}\cos\frac{\pi}{3}$

$= \frac{e^{\frac{3}{4}}}{2}$

16. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to :

(1) 4 (2) 2 (3) 3 (4) 1

Sol. **(3)**

$96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{2^2\pi}{33}\cos\frac{2^3\pi}{33}\cos\frac{2^4\pi}{33}$

$\because$ cos A cos 2A cos 2^2A …. Cos 2^{n-1} A = $\frac{\sin(2^n A)}{2^n \sin A}$

Here A = $\frac{\pi}{33}$, n = 5

$= \frac{96\sin\left(2^5\frac{\pi}{33}\right)}{2^5\sin\left(\frac{\pi}{33}\right)}$

$= \frac{96\sin\left(\frac{32\pi}{33}\right)}{32\sin\left(\frac{\pi}{33}\right)}$

$= \frac{3\sin\left(\pi-\frac{\pi}{33}\right)}{\sin\left(\frac{\pi}{33}\right)} = 3$

17. Let the complex number z = x + iy be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to :

(1) $\frac{3}{2}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) $\frac{3}{4}$

Sol. **(4)**

z = x + iy

$\frac{(2z-3i)}{2z+i}$ = purely imaginary

Means Re $\left(\frac{2z-3i}{2z+i}\right) = 0$

$\Rightarrow \dfrac{(2z-3i)}{(2z+i)} = \dfrac{2(x+iy)-3i}{2(x+iy)+i}$

$= \dfrac{2x+2yi-3i}{2x+i2y+i}$

$= \dfrac{2x+i(2y-3)}{2x+i(2y+1)} \times \dfrac{2x-i(2y+1)}{2x-i(2y+1)}$

$\text{Re}\left[\dfrac{2z-3i}{2z+i}\right] = \dfrac{4x^2+(2y-3)(2y+1)}{4x^2+(2y+1)^2} = 0$

$\Rightarrow 4x^2 + (2y-3)(2y+1) = 0$

$\Rightarrow 4x^2 + [4y^2 + 2y - 6y - 3] = 0$

$\because x + y^2 = 0 \Rightarrow x = -y^2$

$\Rightarrow 4(-y^2)^2 + 4y^2 - 4y - 3 = 0$

$\Rightarrow 4y^4 + 4y^2 - 4y = 3$

$\Rightarrow y^4 + y^2 - y = \dfrac{3}{4}$

Therefore, correct answer is option (4).

18. If $f(x) = \dfrac{(\tan 1°)x + \log_e(123)}{x\log_e(1234) - (\tan 1°)}$, x > 0, then the least value of $f(f(x)) + f\left(f\left(\dfrac{4}{x}\right)\right)$ is :

(1) 2 (2) 4 (3) 8 (4) 0

Sol. **(2)**

$f(x) = \dfrac{(\tan 1)x + \log 123}{x\log 1234 - \tan 1}$

Let $A = \tan 1,\ \ B = \log 123,\ \ C = \log 1234$

$f(x) = \dfrac{Ax+B}{xC-A}$

$$f(f(x)) = \frac{A\left(\dfrac{Ax+B}{xC-A}\right)+B}{C\left(\dfrac{Ax+B}{CX-A}\right)-A}$$

$= \dfrac{A^2x + AB + xBC - AB}{ACx + BC - ACx + A^2}$

$= \dfrac{x(A^2+BC)}{(A^2+BC)} = x$

$f(f(x)) = x$

$f\left(f\left(\dfrac{4}{x}\right)\right) = \dfrac{4}{x}$

$f(f(x)) + f\left(f\left(\dfrac{4}{x}\right)\right)$

$AM \geq GM$

$x + \dfrac{4}{x} \geq 4$

19. The slope of tangent at any point (x, y) on a curve y = y(x) is $\frac{x^2+y^2}{2xy}, x > 0$. If y (2) = 0, then a value of y (8) is :

(1) $4\sqrt{3}$ (2) $-4\sqrt{2}$ (3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

Sol. **(1)**

$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

y = vx

y(2) = 0

y(8) = ?

$\frac{dv}{dx} = v + x\frac{dv}{dx}$

$v + \frac{xdv}{dx} = \frac{x^2+v^2x^2}{2vx^2}$

$x.\frac{dv}{dx} = \left(\frac{v^2+1}{2v} - v\right)$

$\frac{2vdv}{(1-v^2)} = \frac{dx}{x}$

$-\ln(1-v^2) = \ln x + C$

$\ln x + \ln(1-v^2) = C$

$\ln\left[x\left(1-\frac{y^2}{x^2}\right)\right] = C$

$\ln\left[\left(\frac{x^2-y^2}{x}\right)\right] = C$

$x^2 - y^2 = cx$

y(2) = 0 at x = 2, y = 0

$4 = 2C \Rightarrow C = 2$

$x^2 - y^2 = 2x$

Hence, at x = 8

$64 - y^2 = 16$

$y = \sqrt{48} = 4\sqrt{3}$

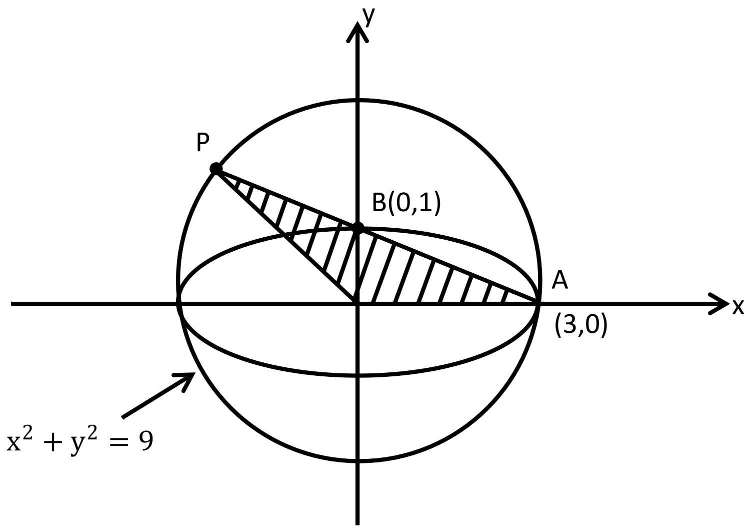
$y(8) = 4\sqrt{3}$

Option (1)

20. Let the ellipse E : $x^2 + 9y^2 = 9$ intersect the positive x-and y-axes at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is $\frac{m}{n}$, where m and n are coprime, then m – n is equal to :

(1) 16 (2) 15 (3) 18 (4) 17

Sol. **(4)**



Equation of line AB or AP is

$$\frac{x}{3}+\frac{y}{1}=1$$

$$x+3y=3$$

$$x=(3-3y)$$

Intersection point of line AP & circle is $P(x_0, y_0)$

$$x^2+y^2=9 \Rightarrow (3-3y)^2+y^2=9$$

$$\Rightarrow 3^2(1+y^2-2y)+y^2=9$$

$$\Rightarrow 5y^2-9y=0 \Rightarrow y(5y-9)=0$$

$$\Rightarrow y=9/5$$

Hence $x=3(1-y)=3\left(1-\frac{9}{5}\right)=3\left(\frac{-4}{5}\right)$

$$x=\frac{-12}{5}$$

$$P(x_0, y_0)=\left(\frac{-12}{5}, \frac{9}{5}\right)$$

Area of ΔAOP is $=\frac{1}{2}\times OA\times \text{height}$

Height $=9/5$, $OA=3$

$$=\frac{1}{2}\times 3\times\frac{9}{5}=\frac{27}{10}=\frac{m}{n}$$

Compare both side $m=27, \quad n=10 \quad \Rightarrow m-n=17$

Therefore, correct answer is option-D

SECTION-B

21. Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple in a match, is 840, then the total numbers of persons, who participated in the tournament, is ____.

Sol. **16**

Let number of couples = n

$\therefore {}^{n}C_2 \times {}^{n-2}C_2 \times 2 = 840$

$\Rightarrow n(n-1)(n-2)(n-3) = 840 \times 2$

$= 21 \times 40 \times 2$

$= 7 \times 3 \times 8 \times 5 \times 2$

$n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$

$\therefore n = 8$

Hence, number of persons = 16.

22. The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ is _____.

Sol. **6**

$-6 < n^2 - 10n + 19 < 6$

$\Rightarrow n^2 - 10n + 25 > 0$ and $n^2 - 10n + 13 < 0$

$(n-5)^2 > 0$ $5 - 3\sqrt{2} < n < 5 + 3\sqrt{2}$

$N \in Z - \{5\}$ $n = \{2, 3, 4, 5, 6, 7, 8\}$

…(i) …(ii)

From (i) $\cap$ (ii)

$N = \{2, 3, 4, 5, 6, 8,\}$

Number of values of n = 6

23. The number of permutations of the digits 1, 2, 3, …., 7 without repetition, which neither contain the string 153 nor the string 2467, is _____.

Sol. **4898**

Numbers are 1, 2, 3, 4, 5, 6, 7

Numbers having string (154) = (154), 2, 3, 6, 7 = 5!

Numbers having string (2467) = (2467), 1,3, 5 = 4!

Number having string (154) and (2467)

= (154), (2467) = 2!

Now n (154 $\cup$ 2467) = 5! + 4! – 2!

= 120 + 24 – 2 = 142

Again total numbers = 7! = 5040

Now required numbers = n (neither 154 nor 2467)

= 5040 – 142

= 4898

24. Let f: (–2, 2) $\to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x[x] & , -2 < x < 0 \\ (x-1)[x], & 0 \le x < 2 \end{cases}$$

where [x] denotes the greatest integer function. If m and n respectively are the number of points in (–2, 2) at which y = |f(x)| is not continuous and not differentiable, then m + n is equal to ______.

Sol. **4**

$$f(x) = \begin{cases} -2x, & -2 < x < -1 \\ -x, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ x-1, & 1 \le x < 2 \end{cases}$$

Clearly f(x) is discontinuous at x = –1 also non differentiable.

$\therefore$ m = 1

Now for differentiability

$$f'(x) = \begin{cases} -2 & -2 < x < -1 \\ -1 & -1 < x < 0 \\ 0 & 0 < x < 1 \\ -1 & 1 < x < 2 \end{cases}$$

Clearly f(x) is non-differentiable at x = –1, 0, 1

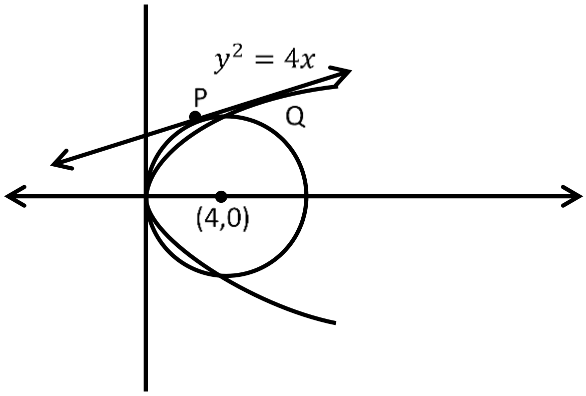
Also, $|f(x)|$ remains same.

$\therefore$ n = 3

$\therefore$ m + n = 4

25. Let a common tangent to the curves $y^2 = 4x$ and $(x-4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to _____ :

Sol. **32**



$y^2 = 4x$

$(x-4)^2 + y^2 = 16$

Let equation of tangent of parabola

$y = mx + 1/m$ ..(1)

Now equation 1 also touches the circle

$$\therefore \left|\frac{4m + 1/m}{\sqrt{1+m^2}}\right| = 4$$

$(4m + 1/m)^2 = 16 + 16m^2$

$16m^4 + 8m^2 + 1 = 16m^2 + 16m^4$

$8m^2 = 1$

$\boxed{m^2 = 1/8}$ $\quad \{m^4 = 0\}(m \to \infty)$

For distinet points consider only $m^2 = 1/8$.

Point of contact of parabola

$P(8, 4\sqrt{2})$

$\therefore PQ = \sqrt{S_1} \Rightarrow (PQ)^2 = S_1$

$= 16 + 32 - 16 = 32$

26. If the mean of the frequency distribution

Class :	0-10	10-20	20-30	30-40	40-50
Frequency :	2	3	x	5	4

is 28, then its variance is ______.

Sol. **151**

C.I.	f	x	f_ix_i	x^2_i
0-10	2	5	10	25
10-20	3	15	45	225
20-30	x	25	25x	625
30-40	5	35	175	1225
40-50	4	45	180	2025

$\bar{x} = \frac{\Sigma f_i x_i}{N}$

$28 = \frac{10 + 45 + 25x + 175 + 130}{14 + x}$

$28 \times 14 + 28x = 410 + 25x$

$\Rightarrow 3x = 410 - 392$

$\Rightarrow x = \frac{18}{3} = 6$

$\therefore$ Variance $= \frac{1}{N} \Sigma f_i x_i^2 - (\bar{x})^2$

$= \frac{1}{20}\ 18700 - (28)^2$

$= 935 - 784 = 151$

27. The coefficient of x^7 in $(1 - x + 2x^3)^{10}$ is ______.

Sol. **960**

$(1 - x + 2x^3)^{10}$

a	b	c
3	7	0
5	4	1
7	1	2

$T_n = \frac{10!}{a!b!c!}(-2x)^b\ (x^3)^c$

$= \frac{10!}{a!b!c!}\ (-2)^b\ x^{b+3c}$

$\Rightarrow b + 3c = 7, a + b + c = 10$

$\therefore$ Coefficient of $x^7 = \frac{10!}{3!7!0!}(-1)^7 + \frac{10!}{5!4!1!}(-1)^4$ **(2)**

$+ \frac{10!}{7!1!2!}(-1)^1 (2)^2$

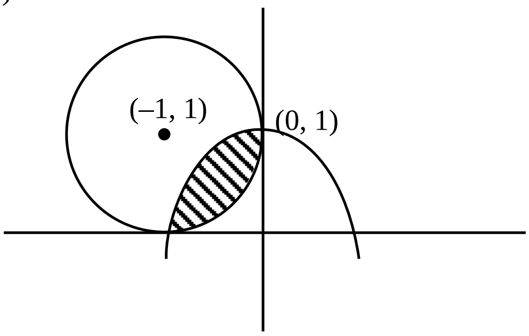
$= -120 + 2520 - 1440 = 960$

28. Let y = p(x) be the parabola passing through the points (–1, 0), (0, 1) and (1, 0). If the area of the region $\{(x, y) : (x+1)^2 + (y-1)^2 \le 1, y \le p(x)\}$ is A, then 12(π – 4A) is equal to _____:

Sol. **16**

There can be infinitely many parabolas through given points.

Let parabola $x^2 = -4a(y-1)$



Passes through (1, 0)

$\therefore\ b = -4a(-1) \Rightarrow a = \frac{1}{4}$

$\therefore\ x^2 = -(y-1)$

Now area covered by parabola $= \int_{-1}^{0} (1 - x^2)dx$

$= \left(x - \frac{x^3}{3}\right)_1^b = (0 - 0) - \left\{-1 + \frac{1}{3}\right\}$

$= \frac{2}{3}$

Required Area = Area of sector – {Area of square – Area covered by Parabola}

$= \frac{\pi}{4} - \left\{1 - \frac{2}{3}\right\}$

$= \frac{\pi}{4} - \frac{1}{3}$

$\therefore\ 12(\pi - 4A) = 12\left[\pi - 4\left(\frac{\pi}{4} - \frac{1}{3}\right)\right]$

$= 12\left[\pi - \pi + \frac{4}{3}\right]$

$= 16$

29. Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$. Then 6a + 5bc is equal to ______.

Sol. **Bouns**

$(2a)^{\ln a} = (bc)^{\ln b} \quad 2a > 0, bc > 0$

$\ln a(\ln 2 + \ln a) = \ln b(\ln b + \ln c)$

$\ln 2 \cdot \ln b = \ln c \cdot \ln a$

$\ln 2 = \alpha, \ln a = x, \ln b = y, \ln c = z$

$\alpha y = xz$

$x(\alpha + x) = y(y + z)$

$\alpha = \frac{xz}{y}$

$x\left(\frac{xz}{y} + x\right) = y(y + z)$

$x^2(z + y) = y^2(y + z)$

$y + z = 0 \text{ or } x^2 = y^2 \Rightarrow x = -y$

$bc = 1 \text{ or } ab = 1$

bc = 1 or ab = 1

(1) if bc = 1 ⇒ $(2a)^{\ln a} = 1$ → a = 1 or a = 1/2

$(a, b, c) = \left(\frac{1}{2}, \lambda, \frac{1}{\lambda}\right), \lambda \neq 1, 2, \frac{1}{2}$

then

6a + 5bc = 3 + 5 = 8

$(II)(a, b, c) = \left(\lambda, \frac{1}{\lambda}, \frac{1}{2}\right), \lambda \neq 1, 2, \frac{1}{2}$

In this situation infinite answer are possible

So, Bonus.

30. The sum of all those terms, of the arithmetic progression 3, 8, 13, …., 373, which are not divisible by 3, is equal to ______.

Sol. **9525**

$A.P: 3, 8, 13.....373$

$T_n = a + (n-1)d$

$373 = 3 + (n-1)5$

$\Rightarrow n = \frac{370}{5}$

$\Rightarrow \boxed{n = 75}$

Now Sum $= \frac{n}{2}[a+l]$

$= \frac{75}{2}[3+373] = 14100$

Now numbers divisible by 3 are,

$3, 18, 33........363$

$363 = 3 + (k-1)15$

$\Rightarrow k-1 = \frac{360}{15} = 24 \Rightarrow \boxed{k=25}$

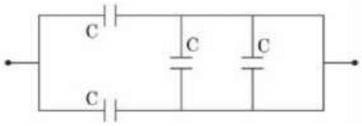
Now, sum $= \frac{25}{2}(3+363) = 4575$ s

$\therefore$ req. sum $= 14100 - 4575$

$= 9525$

SECTION - A

31. The equivalent capacitance of the combination shown is



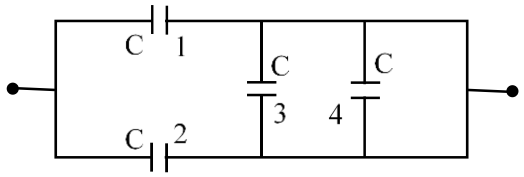
(1) 4C (2) $\frac{5}{3}C$ (3) $\frac{C}{2}$ (4) 2C

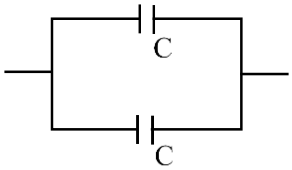
Sol. **(4)**

Capacitor (3) & (4) are short ckt

$\therefore$ C_1 & C_2 are in parallel

$C_{eq.} = C + C = 2C$





32. Match List I with List II :

List I	List II
(A) 3 Translational degrees of freedom	(I) Monoatomic gases
(B) 3 Translational, 2 rotational degrees of freedoms	(II) Polyatomic gases
(C) 3 Translational, 2 rotational and 1 vibrational degrees of freedom	(III) Rigid diatomic gases
(D) 3 Translational, 3 rotational and more than one vibrational degrees of freedom	(IV) Nonrigid diatomic gases

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
(2) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)
(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(4) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Sol. **(1)**

Fact Based

Type of gas	No of degree of freedom
1 Monoatomic 2. Diatomic + rigid	3 (Translational) 3 (Translational + 2 Rotational = 5)
3. Diatomic + non – rigid	3 (Trans) + 2 (Rotational) + 1 (vibrational)
4. Polyatomic	3 (Trans) + 2(Rotational) + more than 1 (vibrational)

33. Given below are two statements :

Statements I : If the number of turns in the coil of a moving coil galvanometer is doubled then the current sensitivity becomes double.

Statements II : Increasing current sensitivity of a moving coil galvanometer by only increasing the number of turns in the coil will also increase its voltage sensitivity in the same ratio

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

Sol. **(3)**

$I = \frac{k\theta}{NBA}$

$C \cdot S = \frac{\theta}{I} = \frac{NBA}{K}$

$N \to 2N \quad C \cdot S \to 2CS$

But $V.S. = \frac{\theta}{V} = \frac{NBA}{KR}$

$N \to 2N C \cdot S \to 2CS$

But $V.S. = \frac{\theta}{v} = \frac{\theta}{IR} = \frac{NBA}{RK}$

As $N \to 2N, R \to 2R$ So V.S = constant

34. Given below are two statements :

Statement I : Maximum power is dissipated in a circuit containing an inductor, a capacitor and a resistor connected in series with an AC source, when resonance occurs

Statement II : Maximum power is dissipated in a circuit containing pure resistor due to zero phase difference between current and voltage.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Sol. **(4)**

Power is more when total impendence of ckt in minimum

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\because X_L = X_C$ (conductor of resonance)

$\therefore Z_{min} = R \quad \therefore V \& I$ in same phase

35. The range of the projectile projected at an angle of 15^o with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45^o with horizontal, then its range will be

(1) $100\sqrt{2}m$ (2) 50 m (3) 100 m (4) $50\sqrt{2}m$

Sol. **(3)**

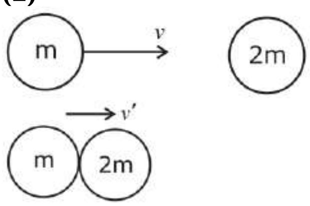
So R = $\frac{u^2 \sin(2 \times 15)}{g} = \frac{u^2}{2g} =$ So $\Rightarrow \frac{u^2}{g} = 100$

$R' = \frac{u^2 \sin(2 \times 45)}{g} = \frac{u^2}{g} = 100\,m$

36. A particle of mass m moving with velocity ν collides with a stationary particle of mass 2m. After collision, they stick together and continue to move together with velocity

(1) $\frac{\nu}{2}$ (2) $\frac{\nu}{3}$ (3) $\frac{\nu}{4}$ (4) ν

Sol. **(2)**



pi = pf $\Rightarrow$ mv + 2m (0) = 3m (v')

$v' = \frac{v}{3}$

37. Two satellites of masses m and 3m revolve around the earth in circular orbits of radii r & 3 r respectively. The ratio of orbital speeds of the satellites respectively is

(1) 3 : 1 (2) 1 : 1 (3) $\sqrt{3}:1$ (4) 9 : 1

Sol. **(3)**

$v=\sqrt{\frac{GM}{r}} \Rightarrow v \times \frac{1}{\sqrt{r}}$; M = mass of earth, r = radius of earth

$\frac{v_1}{v_2}=\sqrt{\frac{r_2}{r_1}}=\sqrt{\frac{3r}{r}}=\sqrt{3}$

38. Assuming the earth to be a sphere of uniform mass density, the weight of a body at a depth $d=\frac{R}{2}$ from the surface of earth, if its weight on the surface of earth is 200 N, will be :

(1) 500 N (2) 400 N (3) 100 N (4) 300 N

Sol. **(3)**

mg = 200 N

$g' = g\left(1-\frac{d}{R}\right)=g\left(1-\frac{R}{2\times R}\right)=\frac{g}{2}$

weight = $mg'=\frac{mg}{2}=\frac{200}{2}=100\,N$

39. The de Broglie wavelength of a molecule in a gas at room temperature (300 K) is λ_1. If the temperature of the gas is increased to 600 K, then the de Broglie wavelength of the same gas molecule becomes

(1) $2\lambda_1$ (2) $\frac{1}{\sqrt{2}}\lambda_1$ (3) $\sqrt{2}\lambda_1$ (4) $\frac{1}{2}\lambda_1$

Sol. **(2)**

$\lambda=\frac{h}{\sqrt{3mK(T)}}$

$\frac{\lambda_1}{\lambda_2}=\sqrt{\frac{T_2}{T_1}}$

$\lambda_2=\lambda_1\sqrt{\frac{T_1}{T_2}}$

$=\lambda_1\sqrt{\frac{300}{600}}=\frac{\lambda_2}{\sqrt{2}}$

40. A physical quantity P is given as

$P=\frac{a^2b^3}{c\sqrt{d}}$

The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4% respectively. The percentage error in the measurement of quantity P will be

(1) 14% (2) 13% (3) 16% (4) 12%

Sol. **(2)**

$\frac{dP}{P}\times 100=\left(2\frac{da}{a}+3\frac{db}{b}+\frac{dc}{c}+\frac{1}{2}\frac{d(d)}{d}\right)\times 100$

$= 2\times 1+3\times 2+3+\frac{1}{2}\times 4$

$= 2+6+3+2$

$= 13\%$

41. Consider two containers A and B containing monoatomic gases at the same Pressure (P), Volume (V) and Temperature (T). The gas in A is compressed isothermally to $\frac{1}{8}$ of its original volume while the gas in B is compressed adiabatically to $\frac{1}{8}$ of its original volume. The ratio of final pressure of gas in B to that of gas in A is

(1) 8 (2) 4 (3) $\frac{1}{8}$ (4) $8^{\frac{3}{2}}$

Sol. **(2)**

By Isothermal Process for (A)

$P_1V_1 = P_2V_2$

$PV = P_2\frac{V}{8}$

$P_2 = 8P$

For B adiabatically $\gamma_{mono} = \frac{5}{3}$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$PV^{5/3} = P_2\left(\frac{V}{8}\right)^{5/3}$

$P_2 = (8)^{5/3} P$

$\frac{P_2}{P_1} = \frac{8^{5/3}}{8P} = (8)^{\frac{2}{3}} = 4$

42. Given below are two statements :

Statements I : Pressure in a reservoir of water is same at all points at the same level of water.

Statements II : The pressure applied to enclosed water is transmitted in all directions equally.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statements I and Statements II are false

(2) Both Statements I and Statements II are true

(3) Statements I is true but Statements II is false

(4) Statements I is false but Statements II is true

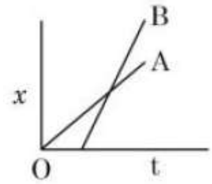
Sol. (2)

Both Statements I and Statements II are true

By Theory

By Pascal law, pressure is equally transmitted to in enclosed water in all direction.

43. The positon-time graphs for two students A and B returning from the school to their homes are shown in figure.



(A) A lives closer to the school

(B) B lives closer to the school

(C) A takes lesser time to reach home

(D) A travels faster than B

(E) B travels faster than A

Choose the correct answer from the options given below :

(1) (A) and (E) only (2) (A), (C) and (E) only

(3) (B) and (E) only (4) (A), (C) and (D) only

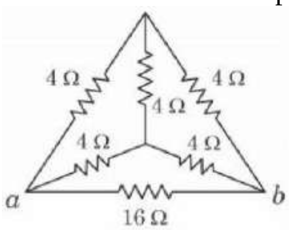
Sol. (1)

(A) and (E) only
Slope of A = V_A
Slope of B = V_B
$(slope)_B > (slope)_A$
$V_B > V_A$
$\therefore t_B < t_A$

44. The energy of an electromagnetic wave contained in a small volume oscillates with
(1) double the frequency of the wave
(2) the frequency of the wave
(3) zero frequency
(4) half the frequency of the wave

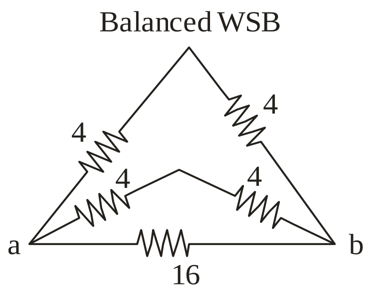
Sol. (1)
double the frequency of the wave
$E = E_0 \sin(wt - kx)$
Energy density = $\frac{1}{2}\varepsilon_0 E_{net}$
$= \frac{1}{2}\varepsilon_0 E_0^2 \sin^2(wt - kx)$
$= \frac{1}{4}\varepsilon_0 E_0^2 (1 - \cos(2wt - 2kx))$

45. The equivalent resistance of the circuit shown below between points a and b is :



(1) 20 Ω (2) 16 Ω (3) 24 Ω (4) 3.2 Ω

Sol. **(4)**
$\frac{1}{R_{ab}} = \frac{1}{16} + \frac{1}{8} + \frac{1}{8}$
$= \frac{1}{R_{ab}} = \frac{1+2+2}{16} = \frac{5}{16}$
$R_{ab} = \frac{16}{5} = 3.2$



46. A carrier wave of amplitude 15 V modulated by a sinusoidal base band signal of amplitude 3 V. The ratio of maximum amplitude to minimum amplitude in an amplitude modulated wave is

(1) 2 (2) 1 (3) 5 (4) $\frac{3}{2}$

Sol. **(4)**
$V_C = 15$
$V_m = 3$
$V_{max} = 15 + 3 = 18$
$Vmin = 15 - 3 = 12$
$Vmax = \frac{18}{12} = \frac{3}{2} = 3 : 2$

47. A particle executes S.H.M. of amplitude A along x-axis. At t = 0, the position of the particle is $x = \frac{A}{2}$ and it moves along positives x-axis. The displacement of particle in time t is x = A $\sin(\omega t + \delta)$, then the value δ will be

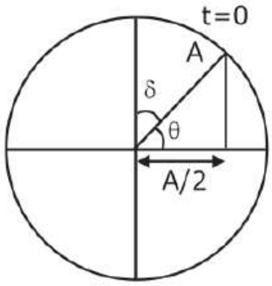
(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

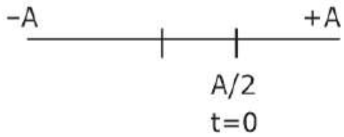
Sol. **(4)**

$\text{Cos } \theta = \frac{A}{2 \times A} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\delta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$





48. The angular momentum for the electron in Bohr's orbit is L. If the electron is assumed to revolve in second orbit of hydrogen atom, them the change in angular momentum will be

(1) $\frac{L}{2}$ (2) zero (3) L (4) 2L

Sol. **(3)**

Angular momentum = $\frac{nh}{2\pi}$

n = 1, $L_1 = \frac{h}{2\pi} = L$

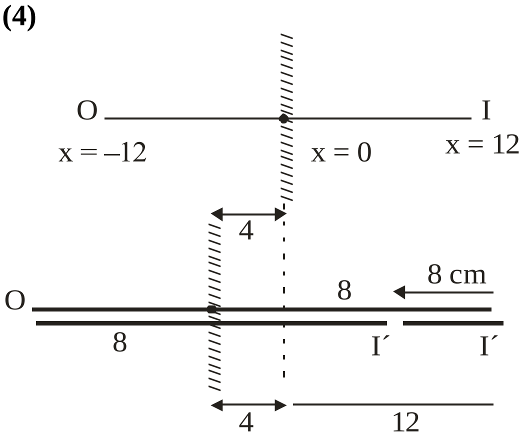
n = 2, $L_2 = \frac{2h}{2\pi} = 2L$

$\Delta L = 2L - L = L$

49. An object is placed at a distance of 12 cm in front of a plane mirror. The virtual and erect image is formed by the mirror. Now the mirror is moved by 4 cm towards the stationary object. The distance by which the position of image would be shifted, will be

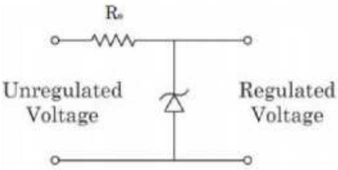
(1) 4 cm towards mirror (2) 8 cm away from mirror
(3) 2 cm towards mirror (4) 8 cm towards mirror

Sol. **(4)**



8 cm towards mirror
Image will be shifted 8 cm towards mirror.

50. A zener diode of power rating 1.6 W is be used as voltage regulator. If the zener diode has a breakdown of 8 V and it has to regulate voltage fluctuating between 3 V and 10 V. The value of resistance R_8 for safe operation of diode will be



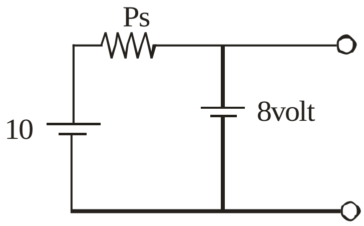
(1) 13.3 Ω (2) 13 Ω (3) 10 Ω (4) 12 Ω

Sol. **(3)**

$I_t = \frac{P_t}{V_t} = \frac{1.6}{8} = 0.2A$

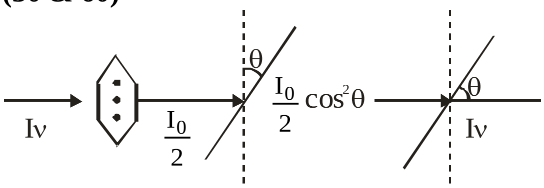
$R_s = \frac{10-8}{I}$

$\boxed{R_s = \frac{2}{0.2} = 10\Omega}$



51. Unpolarised light of intensity 32 Wm^{-2} passes through the combination of three polaroids such that the pass axis of the last polaroid is perpendicular to that of the pass axis of first polaroid. If intensity of emerging light is 3 Wm^{-2}, then the angle between pass axis of first two polaroids is _______°.

Sol. **(30 & 60)**



$I_{net} = 3 = \frac{32}{8}(\sin 2\theta)^2 \qquad = \frac{I_0}{2}\cos^2\theta \sin^2\theta$

$\text{Sin}(2\theta) = \frac{\sqrt{3}}{2} \Rightarrow 2\theta = 60 \ \& \ 120° \qquad = \frac{I_0}{8}(\sin 2\theta)^2$

$\boxed{\theta = 30°}$ & 60°

52. If the earth suddenly shrinks to $\frac{1}{64}$th of its original volume with its mass remaining the same, the period of rotation of earth becomes $\frac{24}{x}h$. The value of x is _________ .

Sol. **(16)**

By AMC

$\frac{2}{5}MR^2\omega_1^2 = \frac{2}{5}M\left(\frac{R}{4}\right)^2 \omega_2$

$\frac{\omega_1}{\omega_2} = \frac{1}{16} = \frac{T_2}{T_1} = \frac{T_2}{24}$

$T_2 = \frac{24}{16} \qquad \boxed{\therefore\ x = 16}$ Ans.

53. Three concentric spherical metallic shells X, Y and Z of radius a, b and c respectively [a < b < c] have surface charge densities $\sigma, -\sigma$ and σ, respectively. The shells X and Z are at same potential. If the radii of X & Y are 2 cm and 3 cm, respectively. The radius of shell Z is __________ cm.

Sol. **(5)**

$q_x = \sigma 4\pi a^2$

$q_y = -\sigma 4\pi b^2$

$q_z = \sigma 4\pi c^2$

Potential of y

$$\frac{q_x}{4\pi\varepsilon_0 a} + \frac{q_y}{4\pi\varepsilon_0 b} + \frac{q_z}{4\pi\varepsilon_0 c} = \frac{q_x}{4\pi\varepsilon_0 c} + \frac{q_y}{4\pi\varepsilon_0 c} + \frac{q_z}{4\pi\varepsilon_0 c}$$

$$\frac{\sigma 4\pi a^2}{a} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c} = 4\pi\sigma \frac{(a^2 - b^2 + c^2)}{C}$$

$c(a - b + c) = a^2 - b^2 + c^2$

$c(a - b) + c^2 = (a + b)(a - b)$

$c(a - b) = (a + b)(a - b)$

$\boxed{c = a + b} = 2 + 3$

$\boxed{c = 5 \text{ cm}}$ Ans.

54. A transverse harmonic wave on a string is given by

$y(x, t) = 5 \sin (6t + 0.003 x)$

where x and y are in cm and t in sec. The wave velocity is _________ ms^{-1}.

Sol. **(20)**

$$v = \frac{w}{k} = \frac{6}{.003 \times 10^2} = \frac{6}{.3} = \frac{60}{3} = 20 \text{ m/s.}$$

55. 10 resistors each of resistance 10 Ω can be connected in such as to get maximum and minimum equivalent resistance. The ratio of maximum and minimum equivalent resistance will be ______.

Sol. **(100)**

$R_{max} \Rightarrow$ in series $\Rightarrow$ $10R = 10 \times 10 = 100\Omega$

$R_{max} \Rightarrow$ in parallel $= \frac{R}{10} = \frac{10}{10} = 1\Omega$

$$\frac{R_{max}}{R_{min}} = \frac{100}{1} = 100 \text{ Ans.}$$

$R_{min} \Rightarrow \frac{100}{1} = 100$ Ans.

56. The decay constant for a radioactive nuclide is $1.5 \times 10^{-5}\ s^{-1}$. Atomic weight of the substance is 60 g $mole^{-1}$, $(N_A = 6 \times 10^{23})$. The activity of 1.0 μg of the substance is _________ $\times 10^{10}$ Bq.

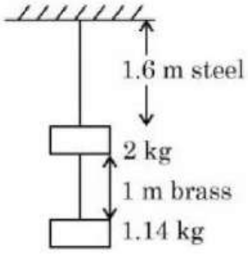
Sol. **(15)**

No. of moles $= \frac{1 \times 10^{-6}}{60} = \frac{10^{-7}}{6}$

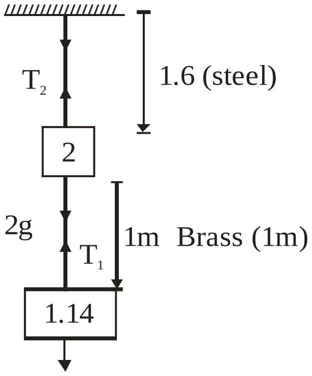
No. of atom $= n(NA) = \frac{10^{-7}}{6} \times 6 \times 10^{23} = 10^{16}$

at $(t = 0)$ $A_0 = No\lambda = 10^{16} \times 1.5 \times 10^{-5} = 15 \times 10^{10}$ Bq

57. Two wires each of radius 0.2 cm and negligible mass, one made of steel and the other made of brass are loaded as shown in the figure. The elongation of the steel wire is _______ × 10^{-6} m. [Young's modulus for steel = 2×10^{11} Nm^{-2} and g = 10 ms^{-2}



Sol. **(20)**



1.14 = 11.4

$T_2 = T_1 + 20 = 20 + 11.4$

$T_2 = 31.4$

$\therefore$ Elongation in steel wire

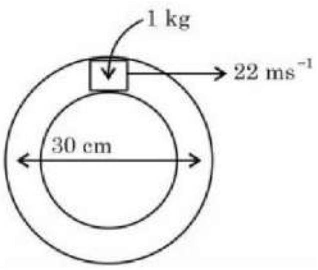
$$\Delta L = \frac{T_2 L}{AY}$$

$$= \frac{31.4 \times 1.6}{\pi(0.2 \times 10^{-2})^2 \times 2 \times 10^{11}}$$

$= 2 \times 10^{-5}$

$\boxed{\Delta L = 20 \times 10^{-6}\ \text{m}}$

58. A closed circular tube of average radius 15 cm, whose inner walls are rough, is kept in vertical plane. A block of mass 1 kg just fit inside the tube. The speed of block is 22 m/s, when it is introduced at the top of tube. After completing five oscillations, the block stops at the bottom region of tube. The work done by the tube on the block is ____ J. (Given : g = 10 m/s^2)



Sol. **(245)**

R_{arg} = 15cm = .15 m

By WET

$W_f + W_{gravity} = \Delta K = Kf - Ki$

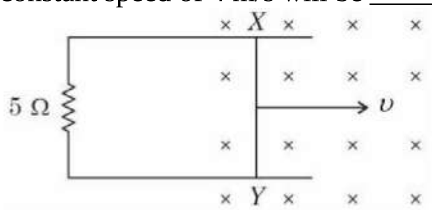
$W_f + 10 \times .3 = 0 - \frac{1}{2} \times 1 \times (22)^2$

$W_f = -3 - \frac{484}{2} = 3 - 242 = -245$

Work by friction = – 245

By NTA (+245)

59. A 1 m long metal rod XY completes the circuit as shown in figure. The plane of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the circuit is $5\,\Omega$, the force needed to move the rod in direction, as indicated, with a constant speed of 4 m/s will be _______ 10^{-3}N.



Sol. **(18)**

$F = I\ell B = \left(\frac{e}{R}\right)\ell B = \frac{(Bv\ell)B\ell}{R} = \frac{B^2\ell^2 v}{R}$

$= \frac{(\bullet 15)^2 \times (1)^2 \times 4}{5} = 180 \times 10^{-4}$

$= 18 \times 10^{-3}$ = **18 Ans.**

60. The current required to be passed through a solenoid of 15 cm length and 60 turns in order to demagnetize a bar magnet of magnetic intensity 2.4×10^3 Am^{-1} is ________A.

Sol. **(6)**

$H = 2.4 \times 10^3$ A/m

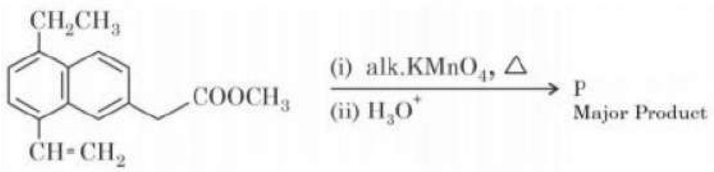
$H = nI = \frac{N}{\ell} I$

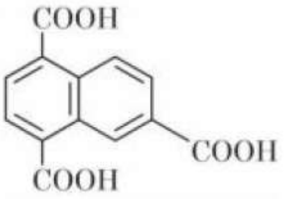
$I = \frac{H\ell}{N} = \frac{2.4 \times 10^3 \times 15 \times 10^{-2}}{60}$

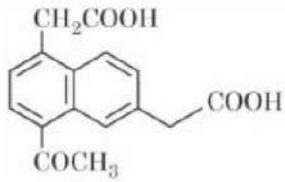
$\boxed{I = 6A}$

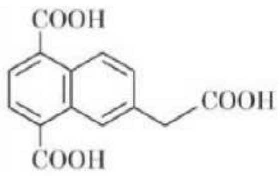
SECTION - A

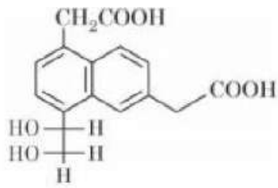
Q.61 The major product 'P' formed in the given reaction is



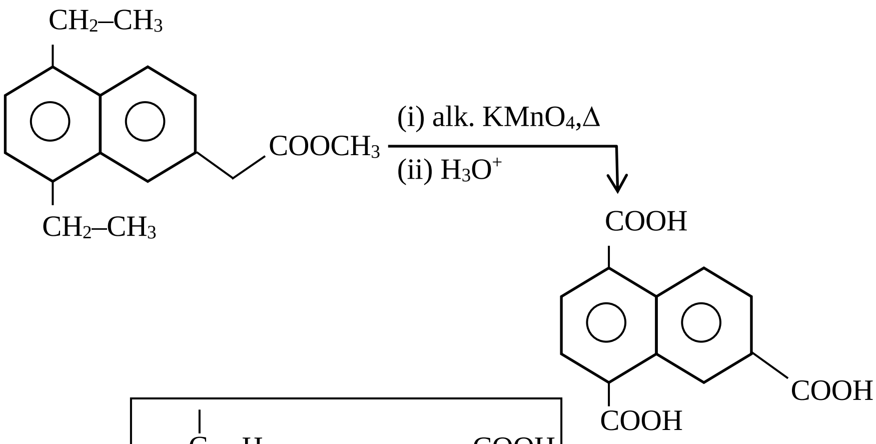
(1) 

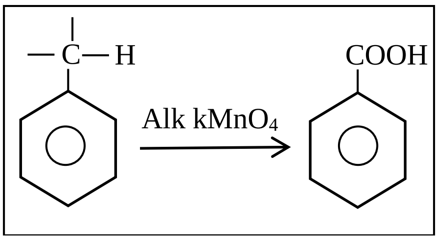
(2) 

(3) 

(4) 

Sol. **(1)**





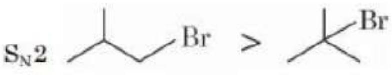
Q.62 Prolonged heating is avoided during the preparation of ferrous ammonium sulphate to

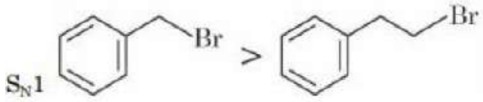
(1) prevent hydrolysis (2) prevent reduction (3) prevent breaking (4) prevent oxidation

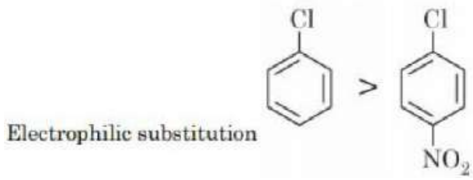
Sol. **(4)**

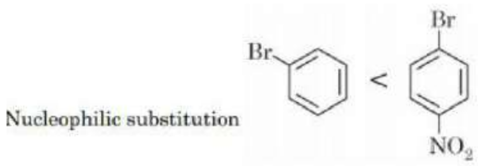
It may oxidise ferrous ion to ferric ions.

Q.63 Identify the correct order of reactivity for the following pairs towards the respective mechanism

(A) 

(B) 

(C) 

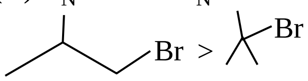
(D) 

Choose the correct answer from the options given below:

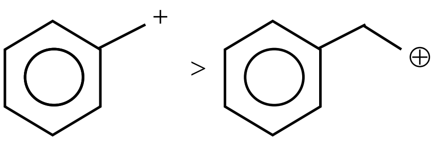
(1) (A), (C) and (D) only
(2) (A), (B) and (D) only
(3) (B), (C) and (D) only
(4) (A), (B), (C) and (D)

Sol. **(4)**

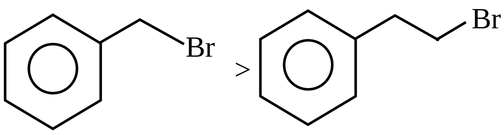
(A) $S_N2 \rightarrow$ for S_N2 Reaction $1^o > 2^o > 3^o$



(B) $S_N1 \rightarrow$ reactivity × Stability of Carbocation formed



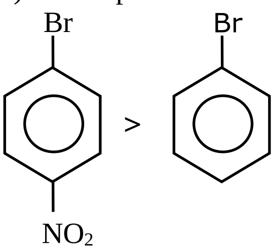
So,



(C) Electrophilic Substitution reaction

$$\text{rate} \times \frac{1}{\text{EWG}}$$

(D) Nucleophilic substitution :- rate × no. of EWG atloched at benzons



Q.64 Given

(A) $2CO(g)+O_2(g) \rightarrow 2CO_2(g)$ $\Delta H_1^o = -x$ KJ mol^{-1}

(B) $C(graphite)+O_2(g) \rightarrow CO_2(g)$ $\Delta H_2^o = -y$ KJ mol^{-1}

The ΔH^o for the reaction

$C(graphite)+ \frac{1}{2}O_2(g) \rightarrow CO(g)$ is

(1) $\frac{x-2y}{2}$
(2) $\frac{x+2y}{2}$
(3) $\frac{2x-y}{2}$
(4) $2y - x$

Sol. **(1)**

$$2CO(g)+O_2(g)\rightarrow 2CO_2(g) \quad \Delta H_1^o = y \text{ kJ / mol} \quad \ldots\ldots\ldots(1)$$

$$C(\text{graphite})+O_2(g)\rightarrow CO_2(g) \quad \Delta H_2^o = -y \text{ kJ / mol} \quad \ldots\ldots\ldots(2)$$

$$C(\text{graphite})+½O_2(g)\rightarrow CO(g) \quad \Delta H_3^o = ?$$

$$\Delta H_3^o = H_2^o - \frac{H_1^o}{2} = -y - \frac{-x}{2}$$

$$\Delta H_3^o = \frac{x}{2} - y = \frac{x-2y}{2}$$

Q.65 Using column chromatography mixture of two compounds 'A' and 'B' was separated. 'A' eluted first, this indicates 'B' has

(1) high R_f, weaker adsorption
(2) high R_f, stronger adsorption
(3) low R_f, stronger adsorption
(4) low R_f, weaker adsorption

Sol. **(3)**

More Polar the compound, the more it will adhere to the adsorbent and the smaller the distance it will travel from baseline, and Lower its R_f value.

B has Low R_f value and strong Adsoption

$$R_f = \frac{\text{distance covered by substance from base line}}{\text{total distance covered by solvent form base line}}$$

Q.66 Lime reacts exothermally with water to give 'A' which has low solubility in water. Aqueous solution of 'A' is often used for the test of CO_2. a test in which insoluble B is formed. If B is further reacted with CO_2 then soluble compound is formed. 'A' is

(1) Quick lime
(2) Slaked lime
(3) White lime
(4) Lime water

Sol. **(2)**

$$CaO + H_2O \rightarrow \underset{A(\text{less soluble})}{Ca(OH)_2}$$

$$Ca(OH)_2 + CO_2 \rightarrow \underset{B(\text{insoluble})}{CaCO_3} + H_2O$$

$$\underset{B}{CaCO_3} + H_2O + CO_2 \rightarrow \underset{\text{Soluble}}{Ca(HCO_3)_2}$$

Q.67 Match list I with list II

List I Industry		List II Waste Generated	
(A)	Steel plants	(I)	Gypsum
(B)	Thermal power plants	(II)	Fly ash
(C)	Fertilizer industries	(III)	Slag
(D)	Paper mills	(IV)	Bio-degradable wastes

Choose the correct answer from the options given below

(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)
(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Sol. **(4)**

Steel plant produces slag from blast furnace. Thermal power plant produces fly ash, Fertilizer industries produces gypsum. Paper mills produces bio degradable waste

Q.68 Suitable reaction condition for preparation of Methyl phenyl ether is

(1) Benzene, MeBr
(2) $PhO^{\ominus}Na^{\oplus}$, MeOH
(3) Ph–Br, $MeO^{\ominus}Na^{\oplus}$
(4) $PhO^{\ominus}Na^{\oplus}$, MeBr

Sol. **(4)**

Williamson;s synthasis :-

$Ph - O^{\ominus}Na^{\oplus} + Me - Br \rightarrow Ph - O - Me + NaBr$

Q.69 The one that does not stabilize 2° and 3° structures of proteins is

(1) H-bonding (2) –S–S–linkage

(3) van der waals forces (4) –O–O–linkage

Sol. **(4)**

Fact

The main forces which stabilize the secondary and tertiary structure of proteins are

→ Hydrogen bonds

→ S – S Linkages

→ vanderwaals force

→ electrostatic force of attraction

Q.70 The compound which does not exist is

(1) $PbEt_4$ (2) BeH_2 (3) NaO_2 (4) $(NH_4)_2BeF_4$

Sol. **(3)**

Sodium superoxide is not stable

Q.71 Given below are two reactions, involved in the commercial production of dihydrogen (H_2). The two reactions are carried out at temperature "T_1" and "T_2", respectively

$C(s)+H_2O(g) \xrightarrow{T_1} CO(g)+H_2(g)$

$CO(g)+H_2O(g) \xrightarrow[\text{catalyst}]{T_2} CO_2(g)+H_2(g)$

The temperatures T_1 and T_2 are correctly related as

(1) $T_1 = T_2$ (2) $T_1 < T_2$ (3) $T_1 > T_2$ (4) $T_1 = 100$ K, $T_2 = 1270$ K

Sol. **(3)**

$T_1 = 1270$ K $T_2 = 673$ K

$T_1 > T_2$ on the basis of data

Q.72 The enthalpy change for the adsorption process and micelle formation respectively are

(1) $\Delta H_{ads} < 0$ and $\Delta H_{mic} < 0$ (2) $\Delta H_{ads} > 0$ and $\Delta H_{mic} < 0$

(3) $\Delta H_{ads} < 0$ and $\Delta H_{mic} > 0$ (4) $\Delta H_{ads} > 0$ and $\Delta H_{mic} > 0$

Sol. **(3)**

Adsorption → Exothermic $(\Delta H_{ads} = -ve)$

Micelle formation → Endothermic $(\Delta H_{mic} = +ve)$

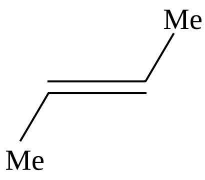
$\Delta H_{ads} < O$ and $\Delta H_{mic} > O$

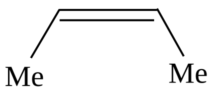
Q.73 The pair from the following pairs having both compounds with net non-zero dipole moment is

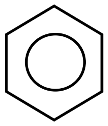
(1) cis-butene, trans-butene (2) Benzene, anisidine

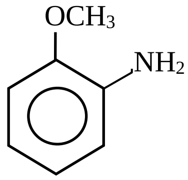
(3) CH_2Cl_2, $CHCl_3$ (4) 1,4-Dichlorohenzene, 1,3-Dichlorobenzene

Sol. **(3)**

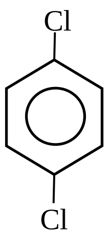
(1) 
$\mu = 0$

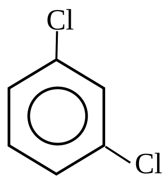

$\mu \neq 0$

(2) 
$\mu = 0$


$\mu \neq 0$

(3) CH_2Cl_2 ($\mu \neq 0$) $CHCl_3$ ($\mu \neq 0$)

(4) 
$\mu = 0$


$\mu \neq 0$

Q.74 Which of the following is used as a stabilizer during the concentration of sulphide ores?
(1) Xanthates (2) Fatty acids (3) Pine oils (4) Cresols

Sol. **4**
Cresol is used as stabilizer

Q.75 Which of the following statements are correct ?
(A) The M^{3+}/M^{2+} reduction potential for iron is greater than manganese
(B) The higher oxidation states of first row d-block elements get stabilized by oxide ion.
(C) Aqueous solution of Cr^{2+} can liberate hydrogen from dilute acid.
(D) Magnetic moment of V^{2+} is observed between 4.4-5.2 BM.
Choose the correct answer from the options given below:
(1) (C), (D) only (B) (B), (C) only (C) (A), (B), (D) only (D) (A), (B) only

Sol. **2**
(A) The M^{3+} / M^{2+} reduction potential for manganese is greater than iron
(B) $E^0_{Fe^{+3}/Fe^{+2}} = +0.77$
$E^0_{Mn^{+3}/Mn^{+2}} = +1.57$
(C) $E^0_{Cr^{+3}/Cr^{+2}} = -0.26$

$$\therefore \quad Cr^{2\oplus} + H^{\oplus} \longrightarrow Cr^{3\oplus} + \frac{1}{2}H_2$$

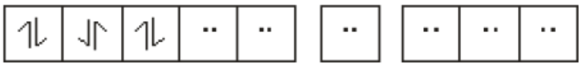
(D) $V^{2\oplus}$ = 3 unpaired electron
Magnetic Moment = 3.87 B.M

Q.76 Given below are two statements :
Statement I : Aqueous solution of $K_2Cr_2O_7$ is preferred as a primary standard in volumetric analysis over $Na_2Cr_2O_7$ aqueous solution.
Statement II : $K_2Cr_2O_7$ has a higher solubility in water than $Na_2Cr_2O_7$
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is false but Statement II is true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are true
(4) Both Statement I and Statement II are false

Sol. **(2)**
(1) $K_2Cr_2O_7$ is used as primary standard. The concentration $Na_2Cr_2O_7$ changes in aq. solution.
(2) It is less soluble than $Na_2Cr_2O_7$

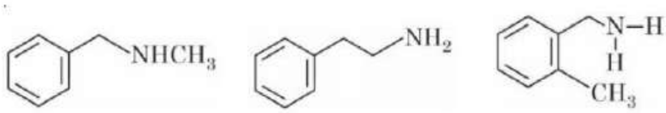
Q.77 The octahedral diamagnetic low spin complex among the following is
(1) $[CoF_6]^{3-}$ (2) $[CoCl_6]^{3-}$ (3) $[Co(NH_3)_6]^{3+}$ (4) $[NiCl_4]^{2-}$

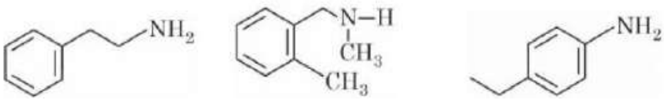
Sol. **(3)**
(1) Paramagnetic, High Spin & Tetrahedral
(2) Paramagnetic, High Spin & Octahedral
(3) Paramagnetic, High Spin & Octahedral
(4) Diamagnetic, Low Spin & Octahedral
$[Co(NH_3)_6]^{3+}, CN = 6$ CN = 6 (Octahedral)
$NH_3 = SFL$
$Co^{+3} = [Ar]3d^6$

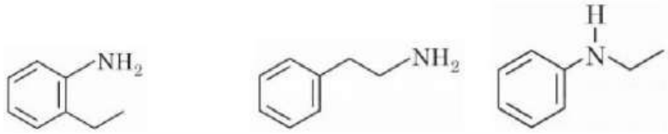


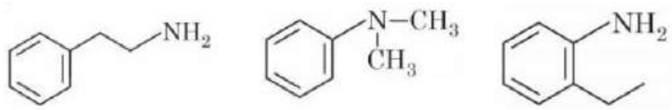
Diamagnetic & Low spin complex

Q.78 Isomeric amines with molecular formula $C_8H_{11}N$ given the following tests
Isomer (P) $\Rightarrow$ Can be prepared by Gabriel phthalimide synthesis
Isomer (Q) $\Rightarrow$ Reacts with Hinsberg's reagent to give solid insoluble in NaOH
Isomer (R) $\Rightarrow$ Reacts with HONO followed by β-naphthol in NaOH to given red dye.
Isomer (P), (Q) and (R) respectively are
(P) (Q) (R)

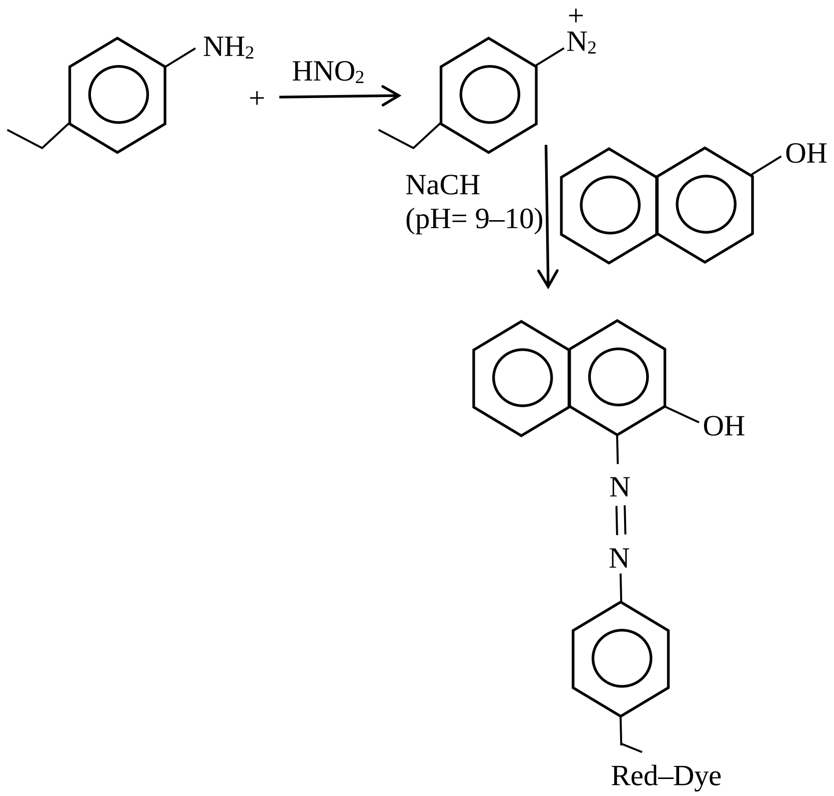
(1) 

(2) 

(3) 

(4) 

Sol. **(2)**
P = Can be prepased by Gabriel phthalimide synthesis it should be i-amine
Q = React with Hinsberg's reagent and insoluble in NaOH it should be 2°–amine
R = React with HNO_2 followed by B–Napthol in NaOH it give red dye it must be Aromatic Amine



Q.79 The number of molecules and moles in 2.8375 litres of O_2 at STP are respectively
(1) 7.527×10^{22} and 0.125 mol (2) 1.505×10^{23} and 0.250 mol
(3) 7.527×10^{23} and 0.125 mol (4) 7.527×10^{22} and 0.250 mol

Sol. **(1)**

Moles of $O_2\left(n_{O_2}\right)=\frac{\text{Volume of } O_2}{22.7}=0.125\,\text{moles}$

Molecules of $O_2=\text{moles}\times N_A$

$= 0.125 \times 6.022 \times 10^{23}$

$= 7.527 \times 10^{22}$ molecules

Ans (1) 7.527×10^{22} and 0.125 mole

Q.80 Match list I with List II

	List I polymer	**List II Type/Class**	
(A)	Nylon-2-Nylon-6	(I)	Thermosetting polymer
(B)	Buna-N	(II)	Biodegradable polymer
(C)	Urea-Formaldehyde resin	(III)	Synthetic rubber
(D)	Dacron	(IV)	Polyester

Choose the correct answer from the options given below:
(1) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(2) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(3) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Sol. **(4)**

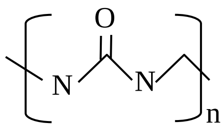
Fact Base

(A) Nylon–2–Nylon–6 $\rightarrow$It is α Biodegradable polymer

(B) Buna – N $\rightarrow$ $\left[CH_2 - CH = CH - CH_2 - CH_2 - \underset{}{\overset{CN}{\overset{|}{C}H}} \right]_n$

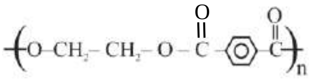
$\rightarrow$ It is a synthetic rubber

(C) Urea – formaldehyde resin



It is a thermos setting polymer

(D) Dacron



It is a polyester.

SECTION - B

Q.81 If the degree of dissociation of aqueous solution of weak monobasic acid is determined to be 0.3, then the observed freezing point will be______________ % higher than the expected/theoretical freezing point. (Nearest integer)

Sol. **30**

For mono basic acid $\rightarrow$ n = 2

$i = 1 + (n-1)\alpha = 1 + (2-1)0.3$

i = 1.3

$$\%\text{increase} = \frac{(\Delta T_f)_{obs} - (\Delta T_f)_{cal}}{(\Delta T_f)_{cal}} \times 100$$

$$= \frac{K_f \times i \times m - K_f \times m}{K_f \times m} \times 100$$

$$= \frac{i-1}{1} \times 100 = 30\%$$

Q.82 In the following reactions, the total number of oxygen atoms in X and Y is___________

$Na_2O + H_2O \rightarrow 2X$

$Cl_2O_7 + H_2O \rightarrow 2Y$

Sol. **5**

$Na_2O + H_2O \rightarrow 2NaOH$

$Cl_2O_7 + H_2O \rightarrow 2HClO_4$

1 + 4 = 5

Q.83 The sum of lone pairs present on the central atom of the interhalogen IF_5 and IF_7 is______

Sol. **1**

IF_5 = 1 lone pair

IF_7 = 0 lone pair

1 + 0 = 1

Q.84 The number of bent-shaped molecule/s from the following is__________

$N_3^-, NO_2^-, I_3^-, O_3, SO_2$

Sol. **3**

N_3^- linear

NO_2^- bent

I_3^- linear

O_3 bent

SO_2 bent

Q.85 The number of correct statement/s involving equilibria in physical from the following is__________

(1) Equilibrium is possible only in a closed system at a given temperature.

(2) Both the opposing processes occur at the same rate.

(3) When equilibrium is attained at a given temperature, the value of all its parameters

(4) For dissolution of solids in liquids, the solubility is constant at a given temperature.

Sol. **3**

(A) is correct

(B) for equilibrium $r_f = r_b$

$\Rightarrow$(B) is correct

(C) at equilibrium the value of parameters become constant of a given temperature and not equal

$\Rightarrow$ (C) is incorrect

(D) for a given solid solute and a liquid solvent solubility depends upon temperature only

$\Rightarrow$ (D) is correct

Q.86 At constant temperature, a gas is at pressure of 940.3 mm Hg. The pressure at which its volume decreases by 40% is__________ mm Hg. (Nearest integer)

Sol. **1567**

$P_{initial}$ = 940.3 mm Hg Vinitial = 100 (Assume)

P_{final} =?

$P_iV_i = P_f V_f$

$940.3 \times 100 = P_f \times 60$

P_f = 1567.16 mm of Hg

P_f = 1567

Q.87 $FeO_4^{2-} \xrightarrow{+2.2V} Fe^{3+} \xrightarrow{+0.70V} Fe^{2+} \xrightarrow{-0.45V} Fe^o$

$E^o_{FeO_4^{2-}/Fe^{2+}}$ is $x \times 10^{-3}$ V. The value of x is___________

Sol. **1825**

$FeO_4{}^{2-} + 3e^{\Theta} \rightarrow Fe^{+3} \quad \Delta G_1$

$Fe^{+3} \quad + e^{\Theta} \rightarrow Fe^{+2} \quad \Delta G_2$

$FeO_4^{-2} \quad + 4e^{\Theta} \rightarrow Fo^{+2} \Delta G_3$

$\Delta G_3 = \Delta G_1 + \Delta G_2$

$(-)4E_3^o F = (-)3 \times 2.2 \times F + (-)1 \times 0.7 \times f$

$4E_3^o = 6.6 + 0.7 = 7.3$

$E_3^o = \frac{7.3}{4} = 1.825 = 1825 \times 10^{-3}$

Q.88 A molecule undergoes two independent first order reactions whose respective half lives are 12 min and 3 min. If both the reactions are occurring then the time taken for the 50% consumption of the reactant is__________ min. (Nearest integer)

Sol. **2**

$k_{eff} = k_1 + k_2$

$$\frac{\ell n^2}{t_{eff}} = \frac{\ell n^2}{t_1} + \frac{\ell n^2}{t_2}$$

$$\frac{1}{t_{eff}} = \frac{1}{12} + \frac{1}{3} = \frac{1+4}{12} = 5/12$$

$$t_{eff} = \frac{12}{5} = 2.4 = 2$$

Q.89 The number of incorrect statement/s about the black body from the following is__________

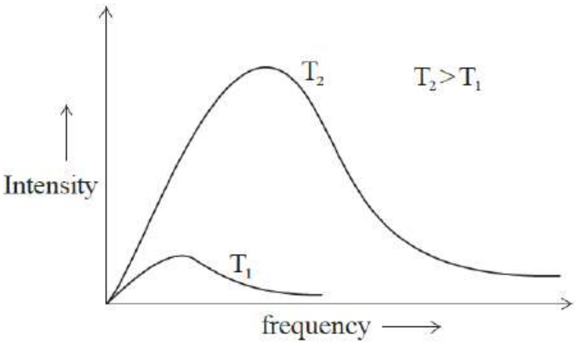
(1) Emit or absorb energy in the form of electromagnetic radiation.

(2) Frequency distribution of the emitted radiation depends on temperature.

(3) At a given temperature, intensity vs frequency curve passes through a maximum value.

(4) The maximum of the intensity vs frequency curve is at a higher frequency at higher temperature compared to that at lower temperature.

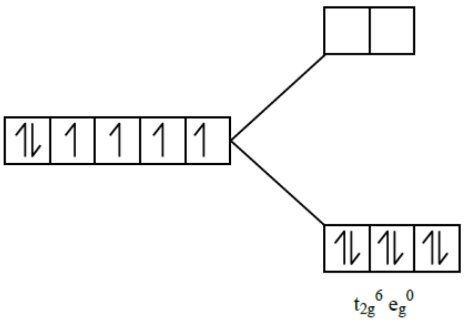
Sol. **0**



Q.90 In potassium ferrocyanide, there are____________ pairs of electrons in the t_{2g} set of orbitals.

Sol. **3**

$K_4[Fe(CN)_6]$



$Fe^{+2} = [Ar]3d^6$

$CN^- = SFL$

t_{2g} contain 6 electron so it become 3 pairs

SECTION-A

1. Let x_1, x_2, ….., x_{100} be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i(x_i - i)$, $1 \le i \le 100$, then the mean of y_1, y_2, ….. y_{100} is :

(1) 10051.50 (2) 10100 (3) 10101.50 (4) 10049.50

Sol. **(4)**

Mean = 200

$$\Rightarrow \frac{\frac{100}{2}(2\times 2+99d)}{100}=200$$

$$\Rightarrow 4+99d=400$$

$$\Rightarrow d=4$$

$$y_i = i(xi-1)$$

$$= i(2+(i-1)4-i)=3i^2-2i$$

$$\text{Mean } = \frac{\sum y_i}{100}$$

$$=\frac{1}{100}\sum_{i=1}^{100}3i^2-2i$$

$$=\frac{1}{100}\left\{\frac{3\times 100\times 101\times 201}{6}-\frac{2\times 100\times 101}{2}\right\}$$

$$=101\left\{\frac{201}{2}-1\right\}=101\times 99.5$$

= 10049.50

2. The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ is :

(1) 10 (2) 9 (3) 8 (4) 12

Sol. **(2)**

$$3\cos^4\theta-5\cos^2\theta-2\sin^6\theta+2=0$$

$$\Rightarrow 3\cos^4\theta-3\cos^2\theta-2\cos^2\theta-2\sin^6\theta+2=0$$

$$\Rightarrow 3\cos^4\theta-3\cos^2\theta+2\sin^2\theta-2\sin^6\theta=0$$

$$\Rightarrow 3\cos^2\theta(\cos^2\theta-1)+2\sin^2\theta(\sin^4\theta-1)=0$$

$$\Rightarrow -3\cos^2\theta\sin^2\theta+2\sin^2\theta(1+\sin^2\theta)\cos^2\theta-1$$

$$\Rightarrow \sin^2\theta\cos^2\theta(2+2\sin^2\theta-3)=0$$

$$\Rightarrow \sin^2\theta\cos^2\theta(2\sin^2\theta-1)=0$$

$$(C1)\sin^2\theta=0\rightarrow 3\,\text{solution};\ \theta=\{0,\pi,2\pi\}$$

$$(C2)\cos^2\theta=0\rightarrow 2\ \text{solution};\ \theta=\left\{\frac{\pi}{2},\frac{3\pi}{2}\right\}$$

$$(C3)\ \sin^2\theta=\frac{1}{2}\rightarrow 4\ \text{solution};\ \theta=\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$$

No. of solution = 9

3. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x\left(\log_e\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$ is equal to :

(1) $\log_e\left(\frac{\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$ (2) $\log_e\left(\frac{2\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$

(3) $\log_e\left(\frac{\sqrt{2}\left(3-\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$ (4) $\log_e\left(\frac{\sqrt{2}\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$

Sol. **(4)**

$I=\int_{-\ln 2}^{\ln 2} e^x\left(\ln\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$

Put $e^x=t\Rightarrow e^x dx=dt$

$I=\int_{1/2}^{2}\ln\left(t+\sqrt{1+t^2}\right)dt$

Applying integration by parts.

$=\left[t\ln\left(t+\sqrt{1+t^2}\right)\right]_{\frac{1}{2}}^{2}-\int_{1/2}^{2}\frac{t}{t+\sqrt{1+t^2}}\left(1+\frac{2t}{2\sqrt{1+t^2}}\right)dt$

$=2\ln\left(2+\sqrt{5}\right)-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\int_{1/2}^{2}\frac{t}{\sqrt{1+t^2}}dt$

$=2\ln\left(2+\sqrt{5}\right)-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\frac{\sqrt{5}}{2}$

$=\ln\left(\frac{\left(2+\sqrt{5}\right)^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{\frac{1}{2}}}\right)-\frac{\sqrt{5}}{2}$

4. Let S = {M = [a_{ij}], a_{ij} ∈ {0, 1, 2}, 1 ≤ i, j ≤ 2} be a sample space and A = {M ∈ S : M is invertible} be an event. Then P(A) is equal to :

(1) $\frac{16}{27}$ (2) $\frac{50}{81}$ (3) $\frac{47}{81}$ (4) $\frac{49}{81}$

Sol. **(2)**

M $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a, b, c, d, $\in\{0,1,2\}$

n(s) = 3^4 = 81

we first bound p $(\bar{A})$

$|m|=0\Rightarrow ad=bc$

$ad=bc=0\Rightarrow$ no. of (a, b, c, d) = $\left(3^2-2^2\right)^2=25$

ad = bc =1 ⇒ no. of (a,b,c,d) = 1^2 = 1

ad = bc = 2 ⇒ no. of (a,b,c,d) = 2^2 = 4

ad = bc = 4 ⇒ no. of (a,b,c,d) = 1^2 = 1

$:P(\bar{A})=\frac{31}{81}\Rightarrow p(A)=\frac{50}{81}$

5. Let f : [2, 4] → ℝ be a differentiable function such that (x $\log_e$ x) f ' (x) + ($\log_e$ x) f(x) + f(x) ≥ 1, x ∈ [2, 4] with f(2) = $\frac{1}{2}$ and f(4) = $\frac{1}{4}$. Consider the following two statements :

(A) : f(x) ≤ 1, for all x ∈ [2, 4]

(B) : f(x) ≥ $\frac{1}{8}$, for all x ∈ [2, 4]

Then,

(1) Only statement (B) is true

(2) Only statement (A) is true

(3) Neither statement (A) nor statement (B) is true

(4) Both the statements (A) and (B) are true

Sol. **(4)**

$x\ell nxf'(x)+\ln xf(x)+f(x)\geq 1, x\in[2,4]$

And f(2) = $\frac{1}{2}$, f(4) = $\frac{1}{4}$

Now xlnx, $\frac{dy}{dx}+(\ln+1)y\geq 1$

$\frac{d}{dx}(y\cdot x\ln x)\geq 1$

$\frac{d}{dx}(f(x).x\ln x)\geq 1$

$\Rightarrow\frac{d}{dx}(x\ln xf(x)-x)\geq 0, x\in[2,4]$

⇒ The function g(x) = x ln x f(x) – x is increasing in [2,4]

And g(2) = 2 ln 2f(2) – 2 = ln 2– 2

g(2) = 4 ln 4f(4) – 4 = ln 4 – 4

= 2(ln 2 – 2)

Now $g(2)\leq g(x)\leq g(4)$

Ln 2 – 2 ≤ x lnx f(x) – x ≤ 2(ln 2– 2)

$\frac{\ln 2-2}{x\ln x}+\frac{1}{\ln x}\leq f(x)\leq\frac{2(\ln 2-2)}{x\ln x}+\frac{1}{\ln x}$

Now for $x\in[2,4]$

$\frac{2(\ell n2-2)}{x\ln x}+\frac{1}{\ell nx}<\frac{2(\ln 2-2)}{2\ln 2}+\frac{1}{\ln 2}=1-\frac{1}{\ln 2}<1$

$\Rightarrow f(x)\leq 1$ for $x\in[2,4]$

$\frac{\ln 2-2}{x\ln x}+\frac{1}{\ln x}\geq\frac{\ln 2-2}{4\ln 4}+\frac{1}{\ln 4}=\frac{1}{8}+\frac{1}{2\ln 2}>\frac{1}{8}$

$\Rightarrow f(x)\geq\frac{1}{8}$ for $x\in[2,4]$

Hence both A and B are true.

6. Let A be a 2 × 2 matrix with real entries such that A' = α A + I, where a ∈ ℝ – {–1, 1}. If det $(A^2 – A) = 4$, then the sum of all possible values of α is equal to :

(1) 0 (2) $\frac{5}{2}$ (3) 2 (4) $\frac{3}{2}$

Sol. **(2)**

$A^T = \alpha A + I$

$A = \alpha A^T + I$

$A = \alpha(\alpha A + I) + I$

$A = \alpha^2 A + (\alpha + 1)I$

$A(1-\alpha^2) = (\alpha+1)I$

$A = \frac{I}{1-\alpha}$ … (1)

$|A| = \frac{1}{(1-\alpha)^2}$ … (2)

$|A^2 - A| = |A||A-1|$ … (3)

$A - I = \frac{I}{I-\alpha} - I = \frac{\alpha}{1-\alpha}I$

$|A-I| = \left(\frac{\alpha}{1-\alpha}\right)^2$ … (4)

Now $|A^2 - A| = 4$

$|A||A-I| = 4$

$\Rightarrow \frac{1}{(1-\alpha)^2}\frac{\alpha^2}{(1-\alpha^2)} = 4$

$\Rightarrow \frac{\alpha}{(1-\alpha)^2} = \pm 2$

$\Rightarrow 2(1-\alpha)^2 = \pm\alpha$

$(C_1)\, 2(1-\alpha)^2 = \alpha$ $(C_2)\, 2(1-\alpha)^3 = -\alpha$

$2\alpha^2 - 5\alpha + 2 = 0 \begin{cases}\alpha_1\\ \alpha_2\end{cases}$ $2\alpha^2 - 3\alpha + 2 = 0$

$\alpha_1 + \alpha_2 = \frac{5}{2}$ $\alpha \notin R$

7. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is :

(1) 5 (2) 7 (3) 8 (4) 6

Sol. **(4)**

$\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$

Feasible region: $x + \frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$

And $x + \frac{7}{2} \neq 1 \Rightarrow x \neq \frac{-5}{2}$

And $\frac{x-7}{2x-3} \neq 0$ and $2x-3 \neq 0$

$\Downarrow$ $\Downarrow$

$x \neq 7$ $x \neq \frac{3}{2}$

Taking intersection: $x \in \left(\frac{-7}{2}, \infty\right) - \left\{-\frac{5}{2}, \frac{3}{2}, 7\right\}$

Now $\log_a b \geq 0$ if $a > 1$ and $b \geq 1$

$a \in (0,1)$ and $b \in (0,1)$

C – I; $x + \frac{7}{2} > 1$ and $\left(\frac{x-7}{2x-3}\right)^2 \geq 1$

$x > -\frac{5}{2}$;$(2x-3)^2 - (x-7)^2 \leq 0$

$(2x-3+x-7)\ (2x-3-x+7) \leq 0$

$(3x-10)(x+4) \leq 0$

$x \in \left[-4, \frac{10}{3}\right]$

Intersection: $x \in \left(\frac{-5}{2}, \frac{10}{3}\right]$

C – II: $x + \frac{7}{2} \in (0,1)$ and $\left(\frac{x-7}{2x-3}\right)^2 \in (0,1)$

$0 < x + \frac{7}{2} < 1$; $\left(\frac{x-7}{2x-3}\right)^2 < 1$

$-\frac{7}{2} < x < \frac{-5}{2}$; $(x-7)^2 < (2x-3)^2$

$x \in (-\infty, -4) \cup \left(\frac{10}{3}, \infty\right)$

No common values of x.

Hence intersection with feasible region

We get $x \in \left(\frac{-5}{2}, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$

Integral value of x are {–2, –1, 0, 1, 2, 3}

No. of integral values = 6

8. For any vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10\,|a_i| < 1$, i = 1, 2, 3, consider the following statements :

(A) : $\max\{|a_1|, |a_2|, |a_3|\} \leq |\vec{a}|$

(B) : $|\vec{a}| \leq 3\max\{|a_1|, |a_2|, |a_3|\}$

(1) Only (B) is true
(2) Both (A) and (B) are true
(3) Neither (A) nor (B) is true
(4) Only (A) is true

Sol. **(2)**

Without loss of generality

Let $|a_1| \leq |a_2| \leq |a_3|$

$|\vec{a}|^2 = |\vec{a}_1|^2 + |\vec{a}_2|^2 + |\vec{a}_3|^2 \ge (a_3)^2$

$\Rightarrow |\vec{a}| \ge |a_3| = \max \{|a_1|, |a_2|, |a_3|\}$

A is true

$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \le |a_3|^2 + |a_3|^2 + |a_3|^2$

$\Rightarrow |\vec{a}|^2 \le 3|a_3|^2$

$\Rightarrow |\vec{a}| \le \sqrt{3}|\vec{a}_3| = \sqrt{3} \max \{|a_1|, |a_2|, |a_3|\}$

$\le 3\max \{|a_1|, |a_2|, |a_3|\}$

(2) is true

9. The number of triplets (x,y,z), where x, y, z are distinct non negative integers satisfying x + y + z = 15, is :

(1) 136 (2) 114 (3) 80 (4) 92

Sol. **(2)**

x + y + z = 15

Total no. solution = ${}^{15+3-1}C_3 = 136$ … (1)

Let x = y $\ne$ z

2x+ z = 15 $\Rightarrow$ z = 15 – 2t

$\Rightarrow r \in \{0,1,2,\ldots 7\} - \{5\}$

$\therefore$ 7 solutions

$\therefore$ there are 21 solutions in which exactly

Two of x, y, z are equal … (2)

There is one solution in which x = y = z … (3)

Required answer = 136 – 21 – 1= 144

10. Let sets A and B have 5 elements each. Let mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is ______.

(1) 36 (2) 40 (3) 32 (4) 38

Sol. **(4)**

ω $A = \{a_1, a_2, a_3, a_4, a_5\}$

$B = \{b_1, b_2, b_3, b_4, b_5\}$

Given, $\sum_{i=1}^{5} ai = 25, \sum_{i=1}^{5} bi = 40$

$$\frac{\sum_{i=1}^{5} a_i^2}{5} - \left(\frac{\sum_{i=1}^{5} a_i}{5}\right)^2 = 12, \frac{\sum_{i=1}^{5} b_i^2}{5} - \left(\frac{\sum_{i=1}^{5} b_i}{5}\right)^2 = 20$$

$$\Rightarrow \sum_{i=1}^{5} a_i^2 = 185, \quad \sum_{i=1}^{5} b_i^2 = 420$$

Now, $C = \{C_1, C_2, \ldots C_{10}\}$

$$\therefore \text{ Mean of C, } \bar{C} = \frac{\left(\sum a_i - 15\right) + \left(\sum b_i - 10\right)}{10}$$

$$\bar{C} = \frac{10+50}{10} = 6$$

$$\therefore \sigma^2 = \frac{\sum_{i=1}^{10} C_i^2}{10} = (\bar{C})^2$$

$$= \frac{\sum(a_i-3)^2 + \sum(b_i-2)^2 +}{10} - (6)^2$$

$$= \frac{\sum a_i^2 + \sum b_i^2 - 6\sum a_i + 4\sum b_i + 65}{10} - 36$$

$$= \frac{185+420-150+160+65}{10} - 36$$

= 32

$\therefore$ Mean + Variance = $\bar{C} + \sigma^2 = 6 + 32 = 38$

11. Area of the region $\{(x, y) : x^2 + (y-2)^2 \le 4, x^2 \ge 2y\}$ is :

(1) $\pi + \frac{8}{3}$ (2) $2\pi + \frac{16}{3}$ (3) $2\pi - \frac{16}{3}$ (4) $\pi - \frac{8}{3}$

Sol. **(3)**

$x^2 + (y-2)^2 \le 2^2$ and $x^2 \ge 2y$

Solving circle and parabola simultaneously:

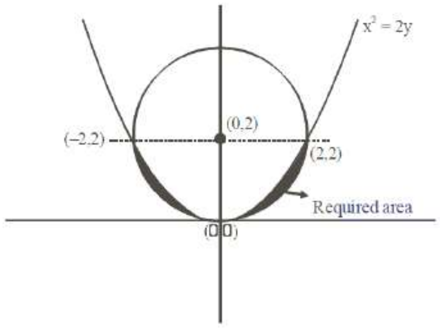
$2y + y^2 - 4y + 4 = 4$

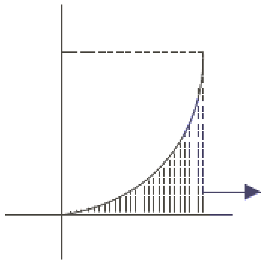
$y^2 - 2y = 0$

y = 0, 2

Put $y = 2$ in $x^2 = 2y \rightarrow x = \pm 2$

$\Rightarrow$ (2, 2) and (–2, 2)





$$= 2 \times 2 - \frac{1}{4} \cdot \pi \cdot 2^2 = 4 - \pi$$

Required area = $2\left[\int_0^2 \frac{x^2}{2} dx - (4-\pi)\right]$

$$= 2\left[\frac{x^3}{6}\Big|_0^2 - 4 + \pi\right]$$

$= 2\left[\frac{4}{3} + \pi - 4\right]$

$= 2\left[\pi - \frac{8}{3}\right]$

$= 2\pi - \frac{16}{6}$

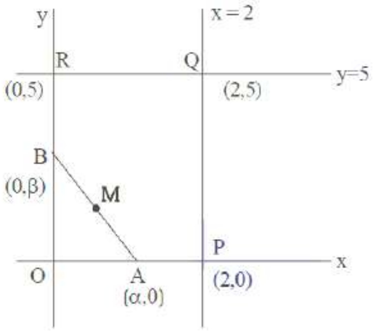
12. Let R be a rectangle given by the line x = 0, x = 2, y = 0 and y = 5. Let A (α, 0) and B (0, β), $\alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4 : l. Then, the mid-point of AB lies on a :

(1) straight line (2) parabola (3) circle (4) hyperbola

Sol. **(4)**

$\frac{ar(OPQR)}{or(OAB)} = \frac{4}{1}$

Let M be the mid-point of AB.



$M(h,k) \equiv \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

$\Rightarrow \frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta} = 4$

$\Rightarrow \frac{5}{2}\alpha\beta = 10 \Rightarrow \alpha\beta = 4$

$\Rightarrow (2h)(2K) = 4$

$\therefore$ Locus of M is xy = 1

Which is a hyperbola.

13. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two places described by $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}, \hat{j} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a}.\vec{b} = 6$, then ordered pair $\left(\theta, |\vec{a} \times \vec{b}|\right)$ is equal to :

(1) $\left(\frac{\pi}{3}, 6\right)$ (2) $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$ (3) $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$ (4) $\left(\frac{\pi}{4}, 6\right)$

Sol. **(4)**

$\vec{n}_1$ and $\vec{n}_2$ are normal vector to the plane $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}, \hat{i} - \hat{k}$ respectively

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} = \lambda|\vec{n}_2 \times \vec{n}_2|$$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = \lambda\left(-2\hat{j} + 2\hat{k}\right)$$

$$\vec{a} \cdot \vec{b} = \lambda|0 + 4 + 2| = 6$$

$$\Rightarrow \lambda = 1$$

$$\vec{\alpha} = -2\hat{j} + 2\hat{k}$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|a||b|}$$

$$\cos\theta = \frac{6}{2\sqrt{2} \times 3} = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

Now $|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |a|^2 |b|^2$

$$36 + |\vec{a} \times b^2| = 8 \times 9 = 72$$

$$|\vec{a} \times b|^2 = 36$$

$$|\vec{a} \times \vec{b}| = 6$$

14. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to :

(1) $\pi - \tan^{-1}\frac{8}{9}$ (2) $-\pi + \tan^{-1}\frac{8}{9}$ (3) $\pi - \tan^{-1}\frac{33}{5}$ (4) $-\pi + \tan^{-1}\frac{33}{5}$

Sol. **(1)**

$$W_1 = z_i i = (5 + 4i)i = -4 + 5i \ \ldots \text{(i)}$$

$$W_1 = z_2(-i) = (3 + 5i)(-i) = 5 - 3i \ \ldots (2)$$

$$W_1 - W_2 = -9 + 8i$$

Principal argument = $\pi - \tan^{-1}\left(\frac{8}{9}\right)$

15. Consider ellipse $E_k : kx^2 + k^2y^2 = 1$, $k = 1, 2, \ldots, 20$. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k. If r_k is the radius of the circle C_k then the value of $\sum_{k=1}^{20} \frac{1}{r_k^{\ 2}}$ is

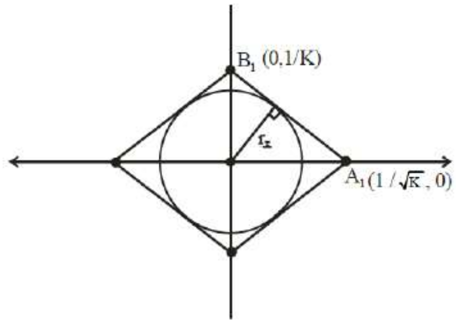
(1) 3320 (2) 3210 (3) 3080 (4) 2870

Sol. **(3)**

$$Kx^2 + K^2y^2 = 1$$

$$\frac{x^2}{1/K}+\frac{y^2}{1/K^2}=1$$

Now



Equation of

$$A_1B_2;\frac{x}{1/\sqrt{K}}+\frac{y}{1/K}=1\Rightarrow\sqrt{K}x+Ky=1$$

$r_K = \perp r$ distance of (0, 0) from line A_1B_1

$$r_K=\left|\frac{(0+0-1)}{\sqrt{K+K^2}}\right|=\frac{1}{\sqrt{K+K^2}}$$

$$\frac{1}{r_K^2}=K+K^2\Rightarrow\sum_{k=1}^{20}\frac{1}{r_K^2}=\sum_{K=1}^{20}\left(K+K^2\right)$$

$$=\sum_{K=1}^{20}K+\sum_{K=1}^{20}K^2$$

$$=\frac{20\times 21}{2}+\frac{20.21.41}{6}$$

= 210 + 10 × 7 × 41

= 210 + 2870

= 3080

16. If equation of the plane that contains the point (–2,3,5) and is perpendicular to each of the planes 2x + 4y + 5z = 8 and 3x – 2y + 3z = 5 is $\alpha x + \beta y + \gamma z + 97 = 0$ then $\alpha + \beta + \gamma$ = :

(1) 15 (2) 18 (3) 17 (4) 16

Sol. **(1)**

The equation of plane through (–2,3,5) is

a(x+2) + b(y – 3) + c (z – 5) = 0

it is perpendicular to 2x + 4y + 5z = 8 & 3x – 2y + 3z = 5

$\therefore$ 2a + 4b + 5c =0

3a – 2b + 3c = 0

$$\therefore\frac{a}{\begin{vmatrix}4 & 5\\ -2 & 3\end{vmatrix}}=\frac{-b}{\begin{vmatrix}2 & 5\\ 3 & 3\end{vmatrix}}=\frac{c}{\begin{vmatrix}2 & 4\\ 3 & -2\end{vmatrix}}$$

$$\Rightarrow\frac{a}{22}=\frac{b}{9}=\frac{c}{-16}$$

$\therefore$ Equation of plane is

22(x + 2) + 9(y – 3) –16(z – 5) = 0

$\Rightarrow$ 22x + 9y – 16z + 97 = 0

Comparing with $\alpha x+\beta y+\gamma x+97=0$

We get $\alpha+\beta+\gamma=22+9-16=15$

17. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events ?

(1) 15 (2) 9 (3) 21 (4) 10

Sol. **(3)**

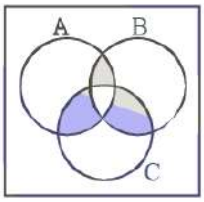
$|A| = 48$

$|B| = 25$

$|C| = 18$

$|A \cup B \cup C| = 60$ [Total]

$|A \cap B \cap C| = 5$



$|A \cap B \cap C| = \sum |A| - \sum |A \cap B| + |A \cap B \cap C|$

$\Rightarrow \sum |A \cap B| = 48 + 25 + 18 + 5 - 60$

$= 36$

No. of men who received exactly 2 medals

$\Rightarrow \sum |A \cap B| - 3|A \cap B \cap C|$

$= 36 - 15$

$= 21$

18. Let y = y(x) be a solution curve of the differential equation. $(1 - x^2y^2)dx = ydx + xdy$. If the line x = 1 intersects the curve y = y(x) at y = 2 and the line x = 2 intersects the curve y = y(x) at y = α, then a value of α is :

(1) $\frac{1+3e^2}{2(3e^2-1)}$ (2) $\frac{1-3e^2}{2(3e^2+1)}$ (3) $\frac{3e^2}{2(3e^2-1)}$ (4) $\frac{3e^2}{2(3e^2+1)}$

Sol. **(1)**

$(1 - x^2y^2)dx = ydx + xdy, y(1) = 2$

$y(2) = \infty$

$dx = \frac{d(xy)}{1-(xy)^2}$

$\int dx = \int \frac{d(xy)}{1-(xy)^2}$

$x = \frac{1}{2}\ln\left|\frac{1+xy}{1-xy}\right| + C$

Put x = 1 and y = 2:

$1 = \frac{1}{2}\ln\left|\frac{1+2}{1-2}\right| + C$

$C = 1 - \frac{1}{2}\ln 3$

Now put x = 2 :

$$2=\frac{1}{2}\ln\left|\frac{1+2\alpha}{1-2\alpha}\right|+1-\frac{1}{2}\ln 3$$

$$1+\frac{1}{2}\ln 3=\frac{1}{2}\left|\frac{1+2\alpha}{1-2\alpha}\right|$$

$$2+\ln 3=\left|\frac{1+2\alpha}{1-2\alpha}\right|$$

$$\left|\frac{1+2\alpha}{1-2\alpha}\right|=3e^{2}$$

$$\frac{1+2\alpha}{1-2\alpha}=3e^{2},-3e^{2}$$

$$\frac{1+2\alpha}{1-2\alpha}=3e^{2}\Rightarrow\alpha=\frac{3e^{2}-1}{2\left(3e^{2}+1\right)}$$

And $\frac{1+2\alpha}{1-2\alpha}=-3e^{2}\Rightarrow\alpha=\frac{3e^{2}+1}{2\left(3e^{2}-1\right)}$

19. Let (α, β, γ) be the image of the point P (2, 3, 5) in the plane 2x + y – 3z = 6. Then α + β + γ is equal to :

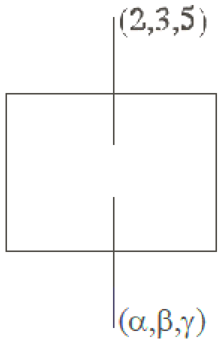
(1) 5 (2) 9 (3) 10 (4) 12

Sol. **(3)**

$$\frac{\alpha-2}{2}=\frac{\beta-3}{1}=\frac{\gamma-5}{-3}=-2\left(\frac{2x2+3-3\times5-6}{2^{2}+1^{2}+1-3^{2}}\right)=2$$

$\frac{\alpha-2}{2}=2$ $\beta-3=2$ $\gamma-5=-6$

$\alpha=6$ $\beta=5$ $\gamma=-1$



$$\alpha+\beta+\gamma=10$$

20. Let f(x) = [x^2 – x] + |–x+[x]|, where x ∈ ℝ and [t] denotes the greatest integer less than or equal to t. Then, f is :

(1) not continuous at x = 0 and x = 1

(2) continuous at x = 0 and x = 1

(3) continuous at x = 1, but not continuous at x = 0

(4) continuous at x = 0, but not continuous at x = 1

Sol. **(3)**

Here f(x) = [x(x – 1)] + {x}

$f\left(0^{+}\right)=-1+0=-1$ $f\left(1^{+}\right)=0+0=0$

$f(0)=0$ $f(1)=0$

$f\left(1^{-}\right)=-1+1=0$

$\therefore$ f(x) is continuous at x = 1, discontinuous at x = 0

SECTION-B

21. The number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{4}}\right)^{680}$ is equal to :

Sol. **(171)**

The number of integral term in the expression of

$\left(3^{\frac{1}{2}}+5^{\frac{1}{4}}\right)^{680}$ is equal to

General term $= {}^{680}C_r\left(3^{\frac{1}{2}}\right)^{680-r}\left(5^{\frac{1}{4}}\right)^{r}$

$= {}^{680}C_r 3^{\frac{680-r}{2}} 5^{\frac{r}{4}}$

Values' s of r, where $\frac{r}{4}$ goes to integer

r = 0, 4, 8, 12, …… 680

All value of r are accepted for $\frac{680-r}{2}$ as well so

No of integral terms = 171.

22. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is True, is equal to _____:

Sol. **(7)**

p	q	r	Pvq	Pvr	(pvq) ∧ (pvr)	qvr	(pvq) ∧ (pvr) → qvr
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

Hence total no of ordered triplets are 7

23. Let $A=\begin{bmatrix}0 & 1 & 2\\ a & 0 & 3\\ 1 & c & 0\end{bmatrix}$, where $a, c \in \mathbb{R}$. If $A^3 = A$ and the positive value of a belongs to the interval (n – 1, n], where $n \in \mathbb{N}$, then n is equal to ______:

Sol. **(2)**

$$A=\begin{bmatrix}0 & 1 & 2\\ a & 0 & 3\\ 1 & c & 0\end{bmatrix}$$

$A^3 = A$

$$A^2=\begin{bmatrix}0 & 1 & 2\\ a & 0 & 3\\ 1 & c & 0\end{bmatrix}\begin{bmatrix}0 & 1 & 2\\ a & 0 & 3\\ 1 & c & 0\end{bmatrix}$$

$$A^2=\begin{bmatrix}a+2 & 2c & 3\\ 3 & a+3c & 2a\\ ac & 1 & 2+3c\end{bmatrix}$$

$$A^3=\begin{bmatrix}a+2 & 2c & 3\\ 3 & a+3c & 2a\\ ac & a & 2+3c\end{bmatrix}\begin{bmatrix}0 & 1 & 2\\ a & 0 & 3\\ 1 & c & 0\end{bmatrix}$$

$$A^3=\begin{bmatrix}2ac+3 & a+2+3c & 2a+4+6c\\ a(a+3c)+2a & 3+2ac & 6+3a+9c\\ a+2+3c & ac+c(2+3c) & 2ac+3\end{bmatrix}$$

Given $A^3 = A$

$2ac + 3 = 0$ … (1) and $a + 2 + 3c = 1$

$a + 1 + 3c = 0$

$a + 1-\frac{9}{2a}=0$

$2a^2 + 2a – 9 = 0$

$f(1) < 0$, $f(2) > 0$

$a\in(1,2]$

$n = 2$

24. For m, n > 0, let $\alpha(m, n) =\int_0^2 t^m(1+3t)^n\,dt$. If $11\alpha(10, 6) + 18\alpha(11, 5) = p\,(14)^6$, then p is equal to _____:

Sol. **(32)**

$$\alpha(m,n)=\int_0^2 t^m(1+3t)^n\,dt$$

If $11\alpha(10,6)+18\alpha(11,5)=p(14)^6$ then P

$$=11\int_0^2 \underset{II}{t^{10}}\,\underset{I}{(1+3t)^6}+10\int^2 t^{11}(1+3t)^5\,dt$$

$$=11\left[(1+3t)^6\cdot\frac{t^{11}}{11}-\int 6(1+3t)^5\cdot 3\frac{t^{11}}{11}\right]_0^2+18\int_0^2 t^{11}(1+3t)^5\,dt$$

$$=\left(t^{11}(1+3t)^6\right)_0^2$$

$$=2^{11}(7)^6$$

$$=2^5(14)^6$$

$$=32(14)^6$$

25. Let S = $S=109+\frac{108}{5}+\frac{107}{5^2}+....+\frac{2}{5^{107}}+\frac{1}{5^{108}}$. Then the value of $(16S-(25)^{-54})$ is equal to _____:

Sol. **(2175)**

$$S=109+\frac{108}{5}+\frac{107}{5^2}......+\frac{1}{5^{108}}$$

$$\frac{S}{5}=\frac{109}{5}+\frac{108}{5^2}......\frac{2}{5^{108}}+\frac{1}{5^{109}}$$

$$\frac{4S}{5}=109-\frac{1}{5}-\frac{1}{5^2}......-\frac{1}{5^{108}}-\frac{1}{5^{109}}$$

$$=109-\left(\frac{1}{5}\frac{\left(1-\frac{1}{5^{109}}\right)}{\left(1-\frac{1}{5}\right)}\right)$$

$$=109-\frac{1}{4}\left(1-\frac{1}{5^{109}}\right)$$

$$=109-\frac{1}{4}+\frac{1}{4}\times\frac{1}{5^{109}}$$

$$s=\frac{5}{4}\left(109-\frac{1}{4}+\frac{1}{4.5^{109}}\right)$$

$$16S=20\times109-5+\frac{1}{5^{108}}$$

$$16S-(25)^{-54}=2180-5=2175$$

26. Let H $H_n:\frac{x^2}{1+n}-\frac{y^2}{3+n}=1, n\in N$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is the length of the latus rectum of H_k, then 21 l is equal to ____:

Sol. **(306)**

$$Hn\Rightarrow\frac{x^2}{1+n}-\frac{y^2}{3+n}=1$$

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{3+n}{1+n}}=\sqrt{\frac{2n+4}{n+1}}$$

$$e=\sqrt{\frac{2n+4}{n+1}}$$

n = 48 (smallest even value for which $e\in Q$)

$$e=\frac{10}{7}$$

$a^2=n+1$ $\quad$ $b^2=n+3$

$=49$ $\quad$ $=51$

l =length of LR = $\frac{2b^2}{a}$

$$L=2\cdot\frac{51}{7}$$

$$1=\frac{102}{7}$$

$$21\ell=306$$

27. The mean of the coefficients of x, x^2, …. x^7 in the binomial expansion of $(2 + x)^9$ is _____:

Sol. **2736**

Coefficient of x = ${}^9C_1 2^8$

Coef. $x^2 = {}^9C_2 2^7$

Coef. $x^7 = {}^9C_7 \cdot 2^2$

Mean = $\dfrac{{}^9C_1 \cdot 2^8 + {}^9C_2 \cdot 2^7 \ldots + {}^9C_7 \cdot 2^2}{7}$

$= \dfrac{(1+2)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2^1 - {}^9C_9}{7}$

$= \dfrac{3^9 - 2^9 - 18 - 1}{7}$

$= \dfrac{19152}{7} = 2736$

28. If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\dfrac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to _____:

Sol. **(51)**

$x^2 - 7x - 1 = 0 <^{a}_{b}$

By newton's theorem

$S_{n+2} - 7S_{n+1} - S_n = 0$

$S_{21} - 7S_{20} - S_{19} = 0$

$S_{20} - 7S_{19} - S_{18} = 0$

$S_{19} - 7S_{18} - S_{17} = 0$

$\dfrac{S_{21} + S_{17}}{S_{19}} = \dfrac{S_{21} + \left(S_{19} - 7S_{18}\right)}{S_{19}}$

$= \dfrac{50S_{19} + \left(S_{21} - 7S_{20}\right)}{S_{19}}$

$= 51 \cdot \dfrac{S_{19}}{S_{19}} = 51$

29. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat, is _______.

Sol. **(44)**

Derangement of 5 students

$D_5 = 5!\left(1 - \dfrac{1}{1!} + \dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - \dfrac{1}{5!}\right)$

$= 120\left(\dfrac{1}{2} - \dfrac{1}{6} + \dfrac{1}{24} - \dfrac{1}{120}\right)$

= 60 – 20 + 5 – 1

= 40+ 4

= 44

30. Let a line l pass through the origin and be perpendicular to the lines

$l_1 : \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R}$ and

$l_2 : \vec{r} = (-\hat{i} + \hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$.

If P is the point of intersection of l and l_1, and Q (α, β, γ) is the foot of perpendicular from P on l_2 , then $9(\alpha + \beta + \gamma)$ is equal to _____:

Sol. **(5)**

Let $\ell = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \gamma(a\hat{i} + b\hat{j} + c\hat{k})$

$= \gamma(a\hat{i} + b\hat{j} + c\hat{k})$

$$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(2-6) - \hat{j}(1-6) + \hat{k}(2-4)$

$= -4\hat{i} - 5\hat{j} - 2\hat{k}$

$\ell = \gamma(-4\hat{i} + 5\hat{j} - 2\hat{k})$

P is intersection of ℓ and ℓ_1

$-4\gamma = 1 + \lambda, 5\gamma = -11 + 2\lambda, -2\gamma = -7 + 3\lambda$

By solving these equation $\gamma = -1, P(4, -5, 2)$

Let Q $(-1 + 2\mu, 2\mu, 1 + \mu)$

$\overrightarrow{PQ} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0$

$-2 + 4\mu + 4\mu + 1 + \mu = 0$

$9\mu = 1$

$\mu = \frac{1}{9}$

$Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$

$9(\alpha + \beta + \gamma) = 9\left(\frac{-7}{9} + \frac{2}{9} + \frac{10}{9}\right)$

= 5

SECTION - A

31. The radii of two planets 'A' and 'B' are 'R' and '4R' and their densities are ρ and $\rho/3$ respectively. The ratio of acceleration due to gravity at their surfaces ($g_A : g_B$) will be :

(1) 1 : 16 (2) 3 : 16 (3) 3 : 4 (4) 4 : 3

Sol. **(3)**

$$g = \frac{4\pi}{3} GR\delta$$

$$g \propto \delta R$$

$$\frac{g_A}{g_B} = \frac{\delta_A R_A}{\delta_B . R_B} = \frac{\delta . R}{\frac{\delta}{3} . 4R} = \frac{3}{4}$$

32. A coin placed on a rotating table just slips when it is placed at a distance of 1 cm from the center. If the angular velocity of the table in halved, it will just slip when placed at a distance of ______ from the centre :

(1) 8 cm (2) 4 cm (3) 2 cm (4) 1 cm

Sol. **(2)**

$fr = m\omega^2 r$

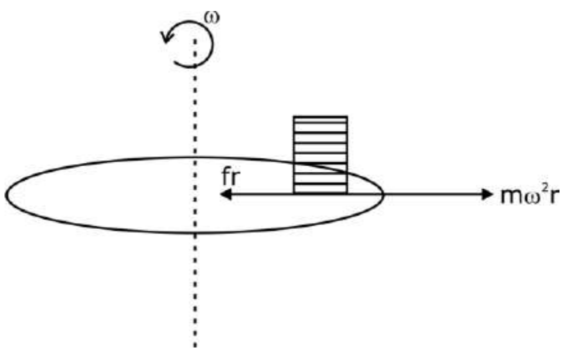
$\mu mg = m\omega^2 r = \text{const.}$

$\omega^2 r = \text{const}$

$\omega_1^2 r_1 = \omega_2^2 r_2$

$\omega^2 (1) = \left(\frac{\omega}{2}\right)^2 r_2$

$r_2 = 4\text{cm}$

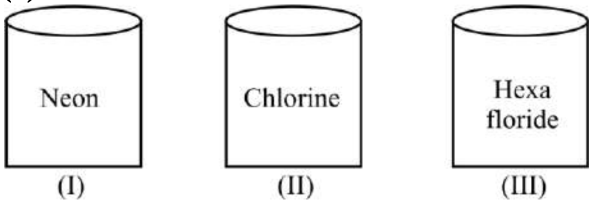


33. Three vessels of equal volume contain gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and third contains uranium hexafluoride (polyatomic). Arrange these on the basis of their root mean square speed (v_{rms}) and choose the correct answer from the options given below :

(1) V_{rms} (mono) > v_{rms} (dia) > v_{rms} (poly) (2) V_{rms} (dia) < v_{rms} (poly) < v_{rms} (mono)

(3) V_{rms} (mono) < v_{rms} (dia) < v_{rms} (poly) (4) V_{rms} (mono) = v_{rms} (dia) = v_{rms} (poly)

Sol. **(1)**



VRMS = $\sqrt{\frac{\gamma RT}{m}}$ ....(1)

$\gamma = 1 + \frac{2}{f}$ so $r_{monochromic} > r_{diatomic} > r_{poly}$.

$V_{mono} > V_{diatomic} > V_{poly}$. Ans.(1)

34. Two radioactive elements A and B initially have same number of atoms. The half life of A is same as the average life of B. If λ_A and λ_B are decay constants of A and B respectively, then choose the correct relation from the given options.

(1) $\lambda_A = 2\lambda_B$ (2) $\lambda_A = \lambda_B$ (3) $\lambda_A \ln 2 = \lambda_B$ (4) $\lambda_A = \lambda_B \ln 2$

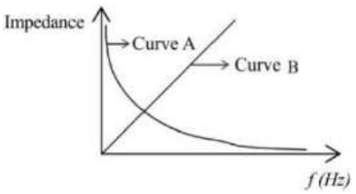
Sol. **(4)**

	A	B	
t = 0	N_0	N_0	$T \rightarrow$ half life
			$\tau \rightarrow$ ang life

$T_A = \tau_B \rightarrow$ given in question

Now $\frac{\ln(2)}{\lambda_A} = \frac{1}{\lambda_B} \Rightarrow \boxed{\lambda_A = \lambda_B \cdot \ln(2)}$

35.



As per the given graph, choose the correct representation for curve A and curve B.
{Where X_C = reactance of pure capacitive circuit connected with A.C. source
X_L = reactance of pure inductive circuit connected with A.C. source
R = impedance of pure resistive circuit connected with A.C. source.
Z = impedance of the LCR series circuit}

(1) A = X_L, B = R (2) A = X_L, B = Z (3) A = X_C, B = R (4) A = X_C, B = X_L

Sol. **(4)**

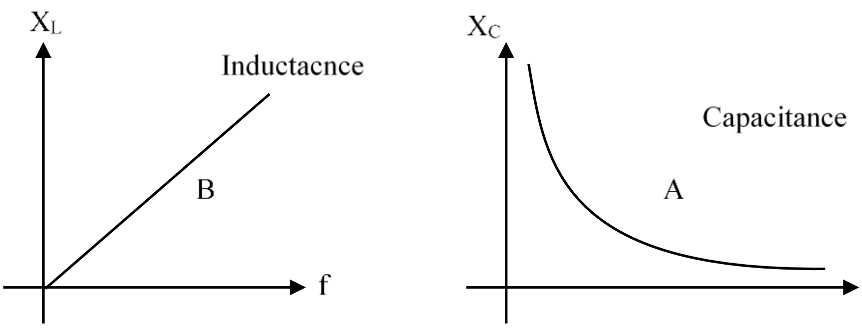
$X_L = W_L = 2\pi fL$

$X_C = \frac{1}{Wc} = \frac{1}{2\pi fc}$

R = const.

A $\rightarrow$ X_C

B $\rightarrow$ X_L



36. A transmitting antenna is kept on the surface of the earth. The minimum height of receiving antenna required to receive the signal in line of sight at 4 km distance from it is $x \times 10^{-2}$ m. The value of x is ______.
(Let, radius of earth R = 6400 km)

(1) 125 (2) 12.5 (3) 1250 (4) 1.25

Sol. **(1)**

$d = \sqrt{2R \cdot h}$

$d^2 = 2Rh$

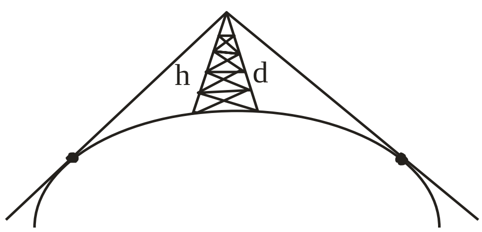
$(4)^2 = 2 \times 6400 \times h$

$\frac{16}{2 \times 6400} = h = \frac{1}{800}$ km

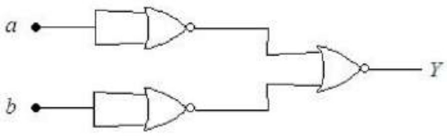
$h = \frac{1000}{800} = \frac{5}{4}$ m

$x \times 10^{-2} = \frac{5}{4}$

$x = \frac{500}{4} = 125$ Ans. Option $\rightarrow$ (1)



37. The logic performed by the circuit shown in figure is equivalent to :



(1) NAND (2) NOR (3) AND (4) OR

Sol. **(3)**

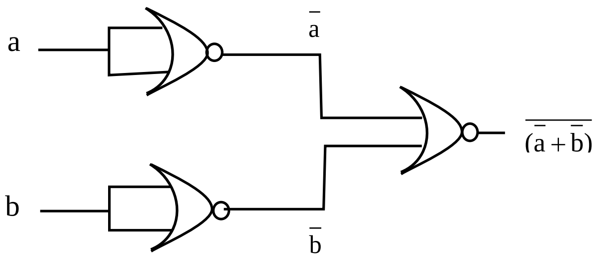
Y = $Y = \overline{(\bar{a}+\bar{b})}$

Y = $Y = (\overline{\overline{a \cdot b}})$

$\boxed{Y = a \cdot b}$

Ans. $\rightarrow$ AND gate

Option $\rightarrow$ 3



38. The electric field in an electromagnetic wave is given as

$\vec{E} = 20\sin\omega\left(t - \frac{x}{c}\right)\vec{j}\,NC^{-1}$

where ω and c are angular frequency and velocity of electromagnetic wave respectively. the energy contained in a volume of 5×10^{-4} m^3 will be (Given $\varepsilon_0 = 8.85 \times 10^{-12} c^2 / Nm^2$)

(1) 88.5×10^{-13} J (2) 17.7×10^{-13} J (3) 8.85×10^{-13} J (4) 28.5×10^{-13} J

Sol. **(3)**

$\vec{E}$ = 20 sin w$\left[t - \frac{x}{c}\right]$

E_o 2_o

Energy density = $\frac{1}{2}\varepsilon_0 E_o^2$

= $\frac{1}{2} \times 8.85 \times 10^{-12} \times 400$

= $200 \times 8.85 \times 10^{-12} \times 5 \times 10^{-4}$

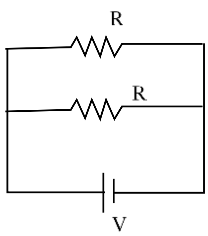
= $8.85 \times 10^{-12} \times 10^{-4} \times 1000$

Energy = 8.85×10^{-13} J option $\rightarrow$ (1)

39. Two identical heater filaments are connected first in parallel and then in series. At the same applied voltage, the ratio of heat produced in same time for parallel to series will be :

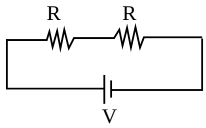
(1) 1 : 2 (2) 4 : 1 (3) 1 : 4 (4) 2 : 1

Sol. **(2)**



$H_1 = \frac{V^2}{(R/2)} t = \frac{2V^2}{R}$ .... (1)

$H_2 = \frac{V^2}{2R}t$ .... (2)



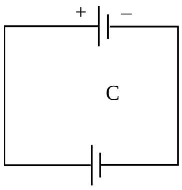
$\frac{H_1}{H_2} = \left(\frac{2V^2t}{R}\right) \times \frac{2R}{V^2t} = \frac{4}{1}$

40. A parallel plate capacitor of capacitance 2 F is charged to a potential V, The energy stored in the capacitor is E_1. The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E_2. The ratio E_2/E_1 is :

(1) 2 : 3 (2) 1 : 2 (3) 1 : 4 (4) 2 : 1

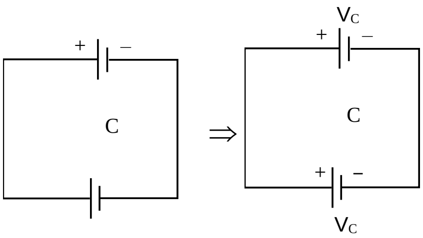
Sol. **(4)**

C = 2F



$E1 = \frac{1}{2}CV^2$ .... (1)

Now



$V_C = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$V_C = \frac{CV + O}{2C} = \frac{V}{2}$

$E_2 = CV_C{}^2 = C \cdot \frac{V^2}{4}$ .... (2)

$\frac{E_2}{E_1} = \frac{\left(\frac{CV^2}{4}\right)}{\left(\frac{CV^2}{2}\right)} = \frac{2}{1}$ option $\rightarrow$ (4)

41. An average force of 125 N is applied on a machine gun firing bullets each of mass 10 g at the speed of 250 m/s to keep it in position. The number of bullets fired per second by the machine gun is :

(1) 25 (2) 5 (3) 100 (4) 50

Sol. **(4)**

F = 125N

$F = \frac{dp}{dt}$ n $\rightarrow$ No. of bullets

$F = \frac{d(nmv)}{dt} = mv\frac{dn}{dt}$

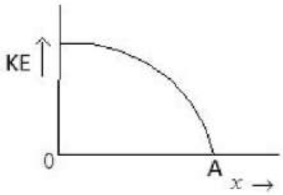
$125 = \frac{10n}{1000} \times 250 \times \frac{dn}{dt}$

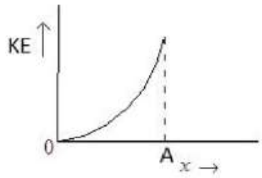
$$\frac{125 \times 1000}{2500} = \frac{dn}{dt}$$

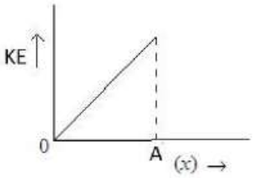
$$\boxed{\frac{dn}{dt} = 50}$$

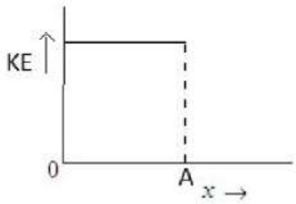
option $\rightarrow$ (4)

42. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by

(1)


(2)


(3)


(4)


Sol. **(1)**

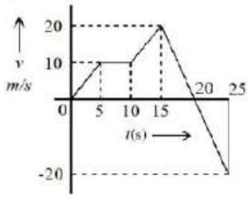
$K \cdot E = T \cdot E - P \cdot E$

$K \cdot E = \frac{1}{2} KA2 - \frac{1}{2} Kx^2$

Graph b/w $K \cdot E$ and x will be parabola

Option $\rightarrow$ (1)

43. From the v – t graph shown, the ratio of distance to displacement in 25 s of motion is :



(1) $\frac{3}{5}$ (2) $\frac{1}{2}$ (3) $\frac{5}{3}$ (4) 1

Sol. **(3)**

Displacement = Area of graph with sign

$$\text{Displacement} = \left(\frac{1}{2} \times 10 \times 5\right) + (10 \times 5) + \left(\frac{1}{2} \times 5 \times 30\right) + \left(\frac{1}{2} \times 5 \times 20\right) - \frac{1}{2}(5)(20)$$

= 25 + 50 + 75 + 50 – 50

= 150 m

Distance $\rightarrow$ Area of graph with positive value

Distance = 25 + 50 + 75 + 50 = 250

$$\frac{\text{Distance}}{\text{Displacement}} = \frac{250}{150} = \frac{5}{3}$$

option $\rightarrow$ (3)

44. On a temperature scale 'X', the boiling point of water is 65° X and the freezing point is-15° X. Assume that the X scale is linear. The equivalent temperature corresponding to -95° X on the Farenheit scale would be :

(1) -63° F (2) -148° F (3) -48° F (4) -112° F

Sol. **(3)**

$$\frac{X_T - X_L}{X_H - X_L} = \frac{T_F - 32}{212 - 32}$$

$$\frac{-95^\circ - (-15^\circ)}{65^\circ - (-15^\circ)} = \frac{T_F - 32}{180}$$

$$\frac{-80^\circ}{80^\circ} = \frac{T_F - 32}{180^\circ}$$

$-180 = T_F - 32$

$T_F = -180 + 32 = -148^\circ$ F   Ans. option → (2)

45. The free space inside a current carrying toroid is filled with a material of susceptibility 2×10^{-2}. The percentage increase in the value of magnetic field inside the toroid will be

(1) 0.2%   (2) 1%   (3) 2%   (4) 0.1%

Sol. **(3)**

$X = 2 \times 10^{-2}$

$\mu r = 1 + x = 1 + 0.02 = 1.02$

Bo → magnetic field due to magnetic material

B_m → magnetic field due to magnetic material

$B_m = \mu_r B_0$

$$\Delta B = \frac{B_m - B_0}{B_0} \times 100 = \frac{\mu_r B_0 - B_0}{B_0} \times 100$$

$$\Delta B\% = \frac{(X+1) - 1}{1} \times 100 = X \times 100$$

$\Delta B\% = 2 \times 10^{-2} \times 100 = 2\%$ Ans. Option (3)

46. The critical angle for a denser-rarer interface is 45°. The speed of light in rarer medium is 3×10^8 m/s. The speed of light in the denser medium is :

(1) 2.12×10^8 m/s   (2) 5×10^7 m/s   (3) 3.12×10^7 m/s   (4) $\sqrt{2} \times 10^8$ m / s

Sol. **(1)**

$$\text{Sin ic} = \frac{\mu_r}{\mu_d} \quad \Rightarrow \sin 45^\circ = \frac{\mu_r}{\mu_d}$$

$$\Rightarrow \frac{\mu_r}{\mu_d} = \sqrt{2} \qquad \text{...........(1)}$$

We know

$$V \propto \frac{1}{\mu} \quad \Rightarrow \frac{V_d}{V_r} = \frac{\mu_r}{\mu_d}$$

$$= \frac{V_d}{3 \times 10^8} = \frac{1}{\sqrt{2}}$$

$$V_d = \frac{3}{\sqrt{2}} \times 10^8 = 3 \times 0.7 \times 10^8$$

$\boxed{V_d = 2.12 \times 10^8 \text{ m/sec}}$ Ans. Option (1)

47. Given below are two statements :

Statements I : Astronomical unit (Au), Parsec (Pc) and Light year (ly) are units for measuring astronomical distances.

Statements II : Au < Parsec (Pc) < ly

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statements I and Statements II are incorrect.

(2) Both Statements I and Statements II are correct.

(3) Statements I is incorrect but Statements II are correct.

(4) Statements I is correct but Statements II are incorrect.

Sol. **(4)**

A.V., Par sec and light year are the unit of distance
Light year $\rightarrow$ distance travelled by light in one year
$1\ ly = 9.5 \times 10^{15}$ m
parcec = 3.262 light year
A.V. = 1.58×10^{-5} light year
A.V. < 1y < Parsec.
Statement I correct and statement II incorrect.

48. The current sensitivity of moving coil galvanometer is increased by 25%.This increase is achieved only by changing in the number of turns of coils and area of cross section of the wire while keeping the resistance of galvanometer coil constant. The percentage change in the voltage sensitivity will be :

(1) +25% (2) -25% (3) -50% (4) Zero

Sol. **(1)**

$\tau = mB$ A = area of coil

$K\theta = IANB$ B = magnetic field

$\frac{\theta}{I} = \frac{ANB}{K}$ Currect senstivity

$1.25\left(\frac{\theta}{I}\right)_2 = \left(\frac{\theta}{I}\right)_1$ .... (1)

$1.25\left[\frac{AN_2B}{K}\right] = \left[\frac{AN_1B}{K}\right]$

$1.25 = \frac{N_1}{N_2} = \frac{5}{4}$ ...... (2)

$\Rightarrow R = \frac{\delta\ell}{a} = \text{const.}$

$\Rightarrow \ell = a$

Voltage sensitivity = $\frac{\theta}{V} = \frac{\theta}{IR} = \frac{\text{Current sensitivity}}{R}$

R = constant

Voltage sensitivity $\propto$ current sensitivity Ans. option $\rightarrow$ (A)

49. A metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V_o. If the same surface is illuminated with radiation of wavelength 2λ, the stopping potential becomes $\frac{V_0}{4}$. The threshold wavelength for this metallic surface will be

(1) $\frac{3}{2}\lambda$ (2) 4λ (3) 3λ (4) $\frac{\lambda}{4}$

Sol. **(3)**

$E = K.E + \phi_0$

Now

$\frac{hc}{\lambda} = ev_0 + \phi_0$ .... (1)

And $\frac{hc}{2\lambda} = \frac{eV_0}{4} + \phi_0$ .... (2)

(2) × 4 .... (1)

$\frac{2hc}{\lambda} - \frac{hc}{\lambda} = 0 + (4\phi_0 - \phi_0)$

$$\frac{hc}{\lambda} = 3\phi_0$$

$$\frac{hc}{\lambda} = 3\frac{hc}{\lambda_0}$$

$$\lambda_0 = 3\lambda$$

50. 1 kg of water at 100°C is converted into steam at 100°C by boiling at atmospheric pressure. The volume of water changes from 1.00×10^{-3} m^3 as a liquid to 1.671 m^3 as steam. The change in internal energy of the system during the process will be

(Given latent heat of vaporisation = 2257 kJ/kg, Atmospheric pressure = 1×10^5 Pa)

(1) + 2476 kJ (2) -2426 kJ (3) -2090 kJ (4) +2090 kJ

Sol. **(4)**

V_1		V_2
Water	$\longrightarrow$	Steam
1kg		100°C
100°C		

Change in volume at constant pressure and temp $\rightarrow$

$\Delta V = V_2 - V_1 = 1.671 - 0.001$

$\Delta V = 1.67$ m^3 ...... (1)

$\Delta Q = \Delta U + w$

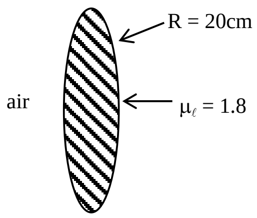
$mL_v = \Delta U + (1.013 \times 10^5)(1.67)$

$\Delta U = (2257 - 170)10^3$

$\Delta U = 2090$ kJ (approx.) Ans. Option $\rightarrow$ 4

51. The radius of curvature of each surface of a convex lens having refractive index 1.8 is 20 cm. The lens is now immersed in a liquid of refractive index 1.5. The ratio of power of lens in air to its power in the liquid will be x : 1. The value of x is _________

Sol. **(4)**



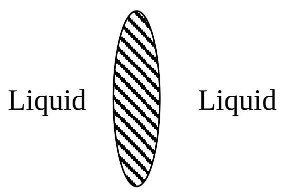
$$\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_m} - 1\right)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$P_1 = \frac{2}{R}\left(\frac{1.8}{1} - 1\right]$$

$$P_1 = \frac{2}{R}(0.8) = \frac{1.6}{R} \quad .... (1)$$

Now,

$$P_2 = \frac{2}{R}\left[\frac{1.8}{1.5} - 1\right]$$

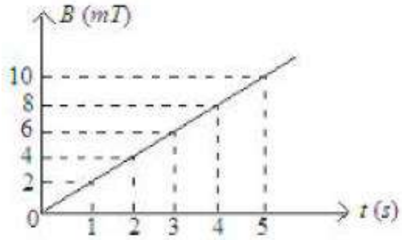


$$P_2 = \frac{2}{R}\left[\frac{0.3}{1.5}\right] = \frac{2}{R} \times \frac{1}{5} = \frac{2}{5R}$$

$$\frac{P_{air}}{P_{liquid}} = \frac{P_1}{P_2} = \frac{\left(\frac{1.6}{R}\right)}{\left(\frac{0.4}{R}\right)} = \frac{4}{1}$$

Ans. $\rightarrow$ 4

52. The magnetic field B crossing normally a square metallic plate of area 4 m^2 is changing with time as shown in figure. The magnitude of induced emf in the plate during t = 2s to t = 4s, is __________ mV.



Sol. **(8)**

$\text{emf} = \frac{d\phi}{dt}$

$\text{Emf} = \frac{dBA}{dt} = \frac{AdB}{dt}$

Emf = 4 · Slope of B.t curve

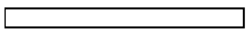
$= 4 \cdot \left[\frac{8-4}{4-2}\right] = 4 \times 2$

$\boxed{\text{Emf} = 8 \text{ Volt}}$

53. The length of a wire becomes l_1 and l_2 when 100 N and 120 N tensions are applied respectively. If 10 l_2 = 11 l_1, the natural length of wire will be $\frac{1}{x}l_1$. Here the value of x is _______ .

Sol. **(2)**

F = kx



ℓ_0 = natural length

$F = \frac{yA}{\ell_0} \cdot x$.

Sol when F = 100 N

$100 = k(\ell_1 - \ell_0)$ ..... (1)

When F = 120N

$120 = K((\ell_1 - \ell_0)$

Given that $10\ell_2 = 11\ell_1$

$\ell_2 = 1.1\,\ell_1$

So $120 = K(1.1\ell_1 - \ell_0)$ .... (2)

Now (2)\(1)

$\frac{120}{100} = \frac{K(1.1\ell_1 - \ell_0)}{K(\ell_1 - \ell_0)}$

$1.2 = \frac{1.1\ell_1 - \ell_0}{\ell_1 - \ell_0}$

$1.2\ell_1 - 1.2\ell_0 = 1.1\ell_1 - \ell_0$

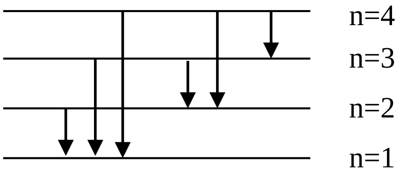
$0.1\ell_1 = 0.2\ell_0$

$\ell_0 = \frac{\ell_1}{2}$ So $\boxed{x = 2}$ Ans.

54. A monochromatic light is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. The frequency of incident light is $x \times 10^{15}$ Hz. The value of x is _______.

(Given $h = 4.25 \times 10^{-15}$ eVs)

Sol. **(3)**



Total emission lines = 6 (given)

So electron absorbed energy and jump from n = 1 to n = 4

$\Delta E\ 13.6 \left[\frac{\ell}{\ell^2} - \frac{1}{4^2}\right]$ ev

$= 13.6 \left[1 - \frac{1}{16}\right]$ ev

$\Delta E = hf$

$12.75 = 4.5 \times 10^{-15}$,

$f = \frac{12.75}{4.25} \times 10^{15} = 3 \times 10^{15}$ Hz

$\boxed{x = 3}$ Ans.

55. A force $\vec{F} = (2 + 3x)\hat{i}$ acts on a particle in the x direction where F is in newton and x is in meter. The work done by this force during a displacement from x = 0 to x = 4 m, is ______ J.

Sol. **(32)**

$\vec{F} = (2 + 3x)i$

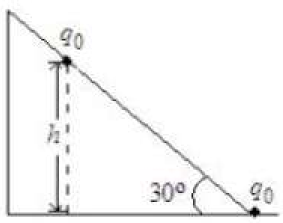
$w = \int_0^4 F.dx = \int_0^4 (2 + 3x).dx$

$w = \left(2x + \frac{3x^2}{2}\right)^4 = (8 + 24)$

$\boxed{w = 32J}$

56. As shown in the figure, a configuration of two equal point charges ($q_0 = +2\,\mu$ C) is placed on an inclined plane. Mass of each point charge is 20 g. Assume that there is no friction between charge and plane. For the system of two point charges to be in equilibrium (at rest) the height $h = x \times 10^{-3}$ m. The value of x is _________.

(Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N\,m^2C^{-2}, g = 10ms^{-2}$)



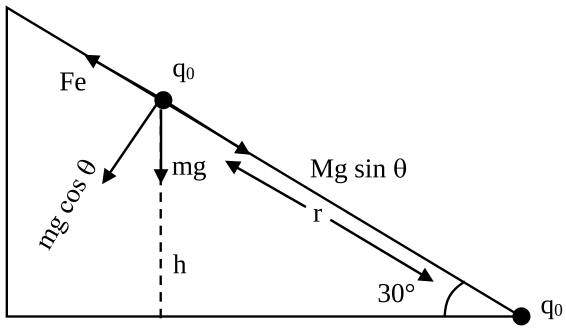
Sol. **(300)**

Point charge on equilibrium is at rest.

So $F_e = mg \sin\theta$

$$\frac{kq_0 \cdot q_0}{r^2} = mg \sin 30°$$

$$\frac{kq_0^2}{\left(\frac{h}{\sin 30°}\right)^2} = \frac{mg}{2}$$



$$\frac{9\times10^9 \times (2\times10^{-6})^2}{4h^2} = \frac{20\times10^{-3}\times10}{2}$$

$$\frac{9\times4\times10^9\times10^{-12}}{4h^2} = 10^{-1}$$

$h^2 = 9\times10^{-2}$

$h = 0.3 \text{ m} = 300 \times 10^{-3} \text{ m}$

$\boxed{x = 300}$ Ans.

57. A solid sphere of mass 500 g and radius 5 cm is rotated about one of its diameter with angular speed of 10 rad s^{-1}. If the moment of inertia of the sphere about its tangent is $x \times 10^{-2}$ times its angular momentum about the diameter. Then the value of x will be _____________ .

Sol. **(35)**

$I_1 = \frac{2}{5}mR^2$

$I_2 = \frac{2}{5} mR^2 + mR^2 = \frac{7}{5}mR^2$

Angular moment about diameter is

$L_{com} = I_1 w = \frac{2}{5}mR^2 w$

Now,

$$\frac{I_2}{L_{com}} = \frac{\frac{7}{5}mR^2}{\frac{2}{5}mR^2 w} = \frac{7}{2}w$$

$$\frac{I_2}{L_{com}} = \frac{7}{2\times10} = \frac{7}{20}$$

Now $\frac{7}{20} = x \times 10^{-2}$

$x = \frac{7}{20} \times 100$

$\boxed{x = 35}$ Ans.

58. The equation of wave is given by $Y = 10^{-2} \sin 2\pi (160t - 0.5x + \pi/4)$

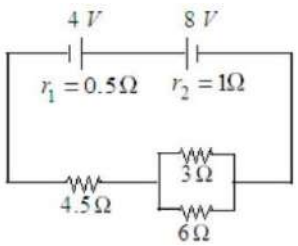
where x and Y are in m and t in s. The speed of the wave is __________ km h^{-1}.

Sol. **(1152)**

$Y = 10^{-2} \sin 2\pi (160t - 0.5x + \pi/4)$

Speed of wave $= \frac{w}{k} = \frac{160}{0.5} = 320$ m/sec $= 320 \times \frac{18}{5} = 1152$ km/h Ans.

59. In the circuit diagram shown in figure given below, the current flowing through resistance 3 Ω is $\frac{x}{3}$ A. The value of x is _________



Sol. **(1)**

Req. = $0.5 + 1 + 4.5 + \left(\frac{3.6}{9}\right)$

Req. = 6 + 2 = 8Ω

$I = \frac{8-4}{8} = \frac{1}{2}$ A = 0.5A

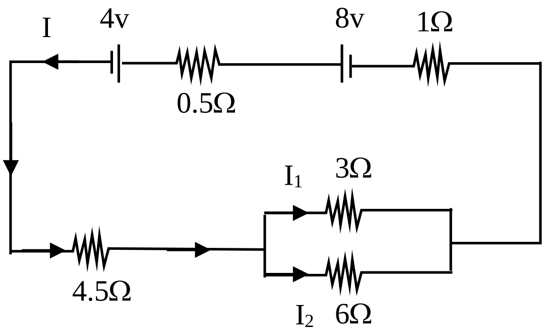
$I_1 : I_2 = \frac{1}{3} : \frac{1}{6}$

$I_1 : I_2 = 2 : 1$

and $I_1 + I_2 = 0.5$ A

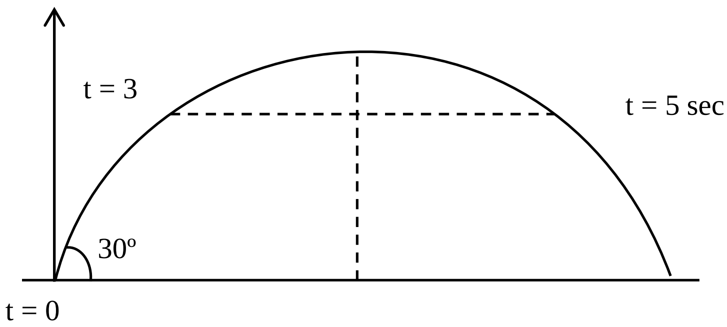
$I_1 = \frac{2}{3} \times 0.5 = \frac{1}{3}$ A

So $\frac{1}{3} = \frac{x}{3} \Rightarrow \boxed{x = 1}$



60. A projectile fired at 30° to the ground is observed to be at same height at time 3s and 5s after projection, during its flight. The speed of projection of the projectile is ________ ms^{-1}.
(Given g = 10 ms^{-2})

Sol. **(80)**
Time of flight = 5 + 3 = 8 sec.



Now, $T = \frac{2u\sin\theta}{g}$

$8 = \frac{2u \cdot \sin 30°}{10}$

$\Rightarrow \boxed{u = 80 \text{ m/sec}}$ Ans.

SECTION - A

61. Which of the following complex has a possibility to exist as meridional isomer?

(1) $[Co(en)_2Cl_2]$ (2) $[Pt(NH_3)_2Cl_2]$

(3) $[Co(en)_3]$ (4) $[Co(NH_3)_3(NO_2)_3]$

Sol. **4**

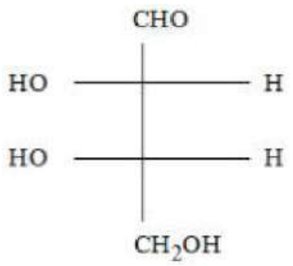
$[MA_3B_3]$ type of compound exists as facial and meridional isomer.

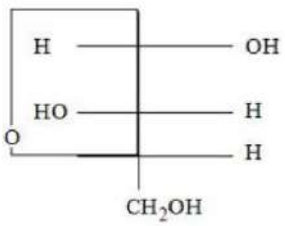
62. L-isomer of tetrose X ($C_4H_8O_4$) gives positive schiff's test and has two chiral carbons. On acetylation. 'X' yields triacetate. 'X' undergoes following reactions

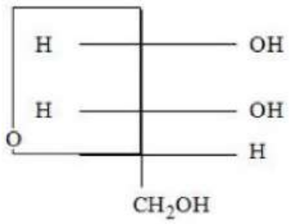
$$'A' \xleftarrow{HNO_3} 'X' \xrightarrow{NaBH_4} 'B'$$

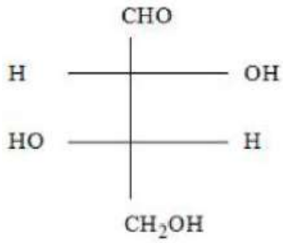
Chiral compound

'X' is

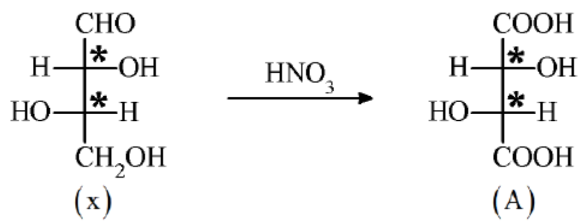
(1)


(2)


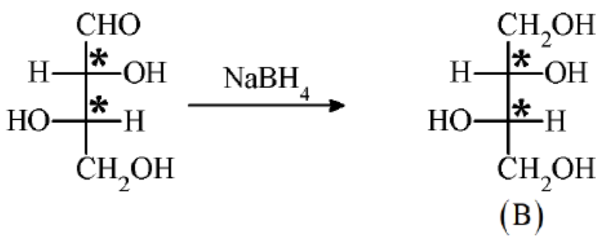
(3)


(4)


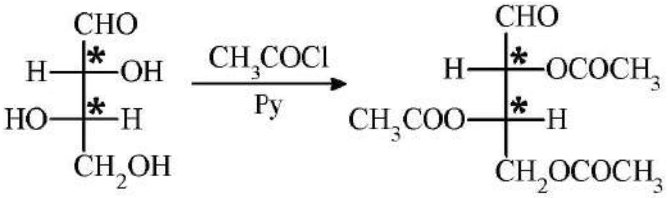
Sol. **4**



L-tetrose with two chiral centre



Optically active



(x) gives positive schiff's test due –CHO group

(x) is L-tetrose.

63. Match list I with list II:

List I	List II
A. K	I. Thermonuclear ractions
B. KCl	II. Fertilizer
C. KOH	III. Sodium potassium pump
D. Li	IV. Absorbent of CO_2

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-II, D-I
(2) A-IV, B-III, C-I, D-II
(3) A-III, B-II, C-IV, D-I
(4) A-IV, B-I, C-III, D-II

Sol. **3**

K^+ –Sodium- Potassium Pump

KCl – Fertiliser

KOH – absorber of CO_2

Li – used in thermonuclear reactions

64. For compound having the formula $GaAlCl_4$, the correct option form the following is

(1) Cl forms bond with both Al and Ga in $GaAlCl_4$

(2) Ga is coordinated with Cl in $GaAlCl_4$

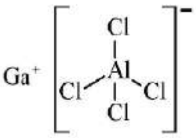
(3) Ga is more electronegative than Al and is present as a cationic part of the salt

(4) Oxidation state of Ga in the salt $GaAlCl_4$ is +3

Sol. **3**

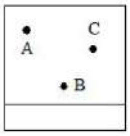
Gallous tetrachloro aluminate $Ga^+AlCl_4^-$

$$2Ga + Ga^+Cl_4^- + 2Al_2Cl_6 \xrightarrow{190^\circ} 4Ga^+AlCl_4^-$$

Structure of $Ga^+AlCl_4^-$ 

Ga is cationic part of salt $GaAlCl_4$.

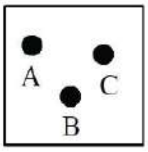
65. Thin layer chromatography of a mixture shows the following observation :



The correct order of elution in the silica gel column chromatography is

(1) B, A, C
(2) C, A, B
(3) A, C, B
(4) B, C, A

Sol. **3**



According to the observation, A is more mobile and interacts with the mobile phase more than C, and C is more drawn to the mobile phase than B.

Hence, the correct order of elution in the silica gel column chromatography is - B < C <A

66. When a solution of mixture having two inorganic salts was treated with freshly prepared ferrous sulphate in acidic medium, a dark brown ring was formed whereas on treatment with neutral $FeCl_3$. it gave deep red colour which disappeared on boiling and a brown red ppt was formed. The mixture contains

(1) $C_2O_4^{2-}$ & NO_3^- (2) SO_3^{2-} & $C_2O_4^{2-}$

(3) CH_3COO^- & NO_3^- (4) SO_3^{2-} & CH_3COO^-

Sol. **3**

$CH_3COO^- + FeCl_3 \rightarrow Fe(CH_3COO)_3$ or $[Fe_3(OH)_2(CH_3COO)_6]^+$

Blood red colour

$\downarrow \Delta$

$Fe(OH)_2(CH_3COO)\downarrow$

Red-brown precipitate

$2NO_3^- + 4H_2SO_4 + 6Fe^{2+} \rightarrow 6Fe^{3+} + 2NO\uparrow + 4SO_4^{2-} + 4H_2O$

$[Fe(H_2O)_6]^{2+} + NO \rightarrow [Fe(H_2O)_5(NO)]^{2+} + H_2O$

Brown

67. The polymer X-consists of linear molecules and is closely packed. It prepared in the presence of triethylaluminium and titranium tetrachloride under low pressure. The polymer X is-

(1) Polyacrylonitrile (2) Polytetrafluoroethane

(3) High density polythene (4) Low density polythene

Sol. **3**

Ethene undergoes addition polymerisation to high density polythene in the presence of catalyst such as $AlEt_3$ and $TiCl_4$ (Ziegler – Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6–7 atmosphere.

68. Match list I with list II

List I Species	**List II Geometry/ Shape**
A. H_3O^+	I. Tetrahedral
B. Acetylide anion	II. Linera
C. NH_4^+	III. Pyramidal
D. ClO_2^-	IV. Bent

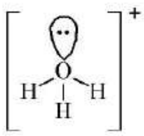
Choose correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II (2) A-III, B-IV, C-II, D-I

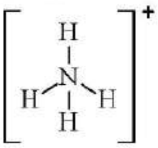
(3) A-III, B-I, C-II, D-IV (4) A-III, B-II, C-I, D-IV

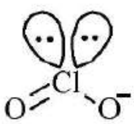
Sol. **4**

Molecule/Ion Hybridisation Shape

H_3O^+ sp^3 Pyramidal 

Acelylide sp linear $\bar{C}\equiv\bar{C}$

NH_4^+ sp^3 tetrahedral 

ClO_2^- sp^3 Bent 

69. Given below are two statement :

Statement I : Methane and steam passed over a heated Ni catalyst produces hydrogen gas

Statement II : Sodium nitrite reacts with NH_4Cl to give H_2O, N_2 and NaCl

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both the statement I and II are incorrect

(2) Statement I is incorrect but statement II is correct

(3) Statement I is correct but statement II is incorrect

(4) Both the statements I and II are correct

Sol. **4**

$$CH_4(g) + \underset{\text{Steam}}{H_2O(g)} \xrightarrow[1270K]{Ni} CO(g) + 3H_2(g)$$

$NaNO_2(aq) + NH_4Cl(aq) \rightarrow N_2(g) + NaCl(aq) + 2H_2O(\ell)$

70. The set which does not have ambidentate ligand (s) is

(1) $C_2O_4^{2-}$, NO_2^-, NCS^-

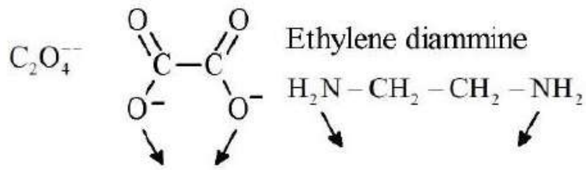
(2) $EDTA^{4-}$, NCS^-, $C_2O_4^{2-}$

(3) NO_2^-, $C_2O_4^{2-}$, $EDTA^{4-}$

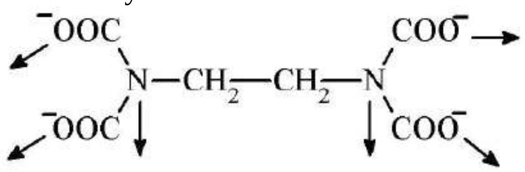
(4) $C_2O_4^{2-}$, ethylene diamine, H_2O

Sol. **4**

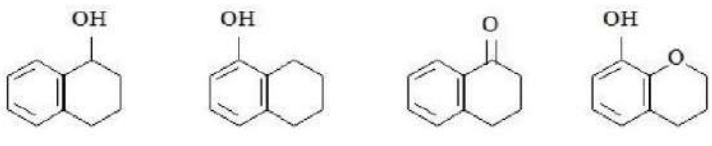
NO_2^- NCS^- are ambidentate ligand



EDTA Ethylene diamine tetra acetate



71. Arrange the following compounds in increasing order of rate of aromatic electrophilic substitution reaction

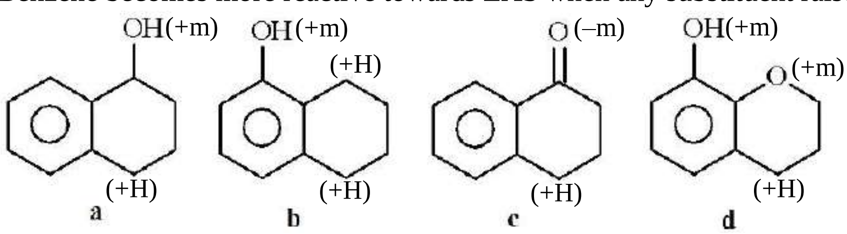


a b c d

(1) c, a, b, d (2) d, b, c, a (3) d, b, a, c (4) b, c, a, d

Sol. **1**

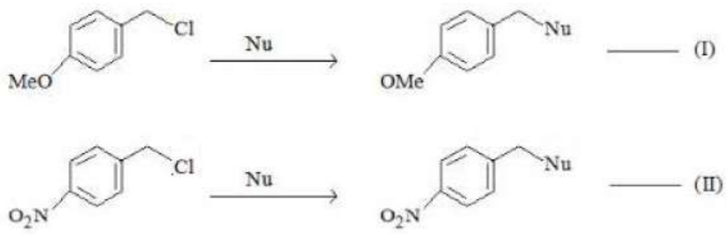
Benzene becomes more reactive towards EAS when any substituent raises the electron density.



Correct order

c < a < b < d

72.



Where Nu = Nucleophile

Find out the correct statement from the options given below for the above 2 reactions.

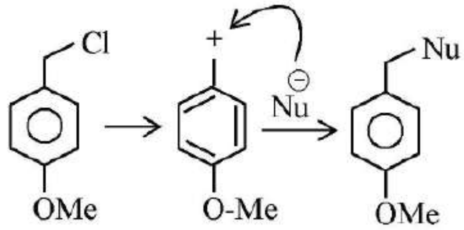
(1) Reaction (I) is of 1st order and reaction (II) is of 2nd order

(2) Reaction (I) and (II) both are 2nd order

(3) Reaction (I) and (II) both are 1st order

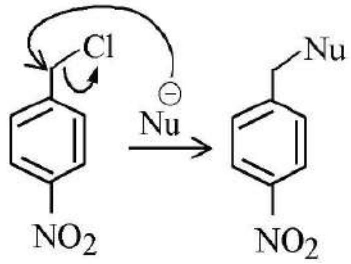
(4) Reaction (I) is of 2nd order and reaction (II) is of 1st order

Sol. **1**



Electron Donating group

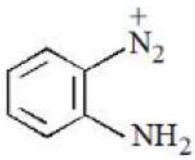
S_N1 Mech. : Ist order

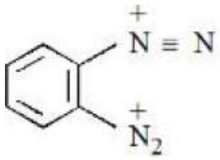


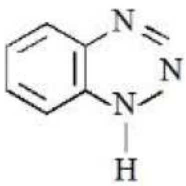
Electron withdrawing group

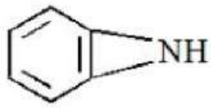
S_N2 Mech: 2nd order

73. o-Phenylenediamine $\xrightarrow{HNO_2}$ 'X' Major Product 'X' is

(1) 

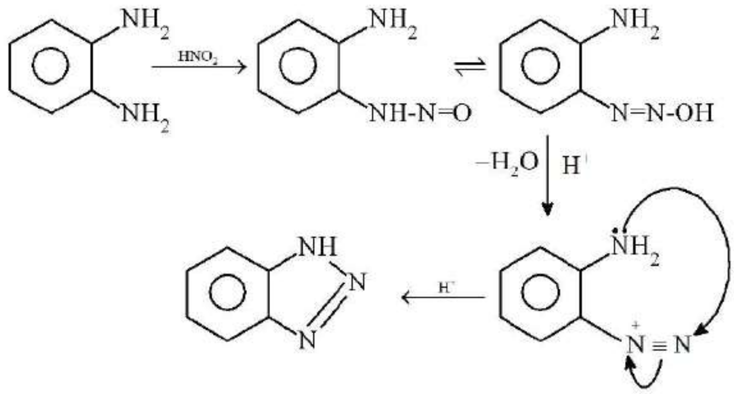
(2) 

(3) 

(4) 

Sol. **3**

o-Phenylenediamine



74. For elements B, C, N, Li, Be, O and F, the correct order of first ionization enthalpy is

(1) B>Li>Be>C>N>O>F (2) Li<Be<B<C<N<O<F

(3) Li<Be<B<C<O<N<F (4) Li<B<Be<C<O<N<F

Sol. **4**

First I.E.

F > N > O > C > Be > B > Li

Li – 520 kJ/mol

Be – 899 kJ/mol

B – 801 kJ/mol

C – 1086 kJ/mol

N – 1402 kJ/mol

O – 1314 kJ/mol

F – 1681 kJ/mol

75. In the extraction process of copper, the product obtained after carrying out the reactions

(i) $2Cu_2S+3O_2 \rightarrow 2Cu_2O+2SO_2$

(ii) $2Cu_2O+Cu_2S \rightarrow 6Cu+SO_2$ is called

(1) Reduced copper (2) Blister copper

(3) Copper matte (4) Copper scrap

Sol. **2**

$2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 3SO_2$

$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$

Blister copper

Due to evolution of SO_2, the solidified copper formed has a blistered look and is referred to as blister copper.

76. 25 mL of silver nitrate solution (1M) is added dropwise to 25 mL of potassium iodide (1.05 M) solution. The ion(s) present in very small quantity in the solution is/are

(1) NO_3^- only (2) Ag^+ and I^- both (3) K^+ only (4) I^- only

Sol. **2**

On adding $AgNO_3$ into KI, AgI will form and solubility of AgI is very low.

So, $[Ag^+]$ and $[I^-]$ will be present in very small quantity.

77. Given below are two statements:

Statement I : If BOD is 4 ppm and dissolved oxygen is 8 ppm, it is a good quality water.

Statement II : If the concentration of zinc and nitrate salts are 5 ppm each, than it can be good quality water.

In the light of the above statements choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but statement II is correct

(2) Statement I is correct but statement II is incorrect

(3) Both the statements I and II are incorrect

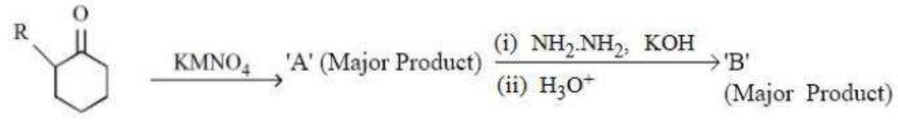
(4) Both the statement I and II are correct

Sol. **4**

Clean water would have BOD value of less than 5 ppm.

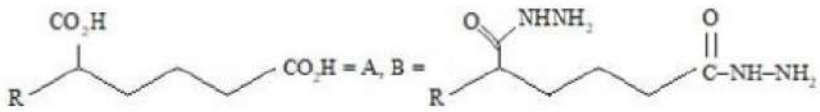
Maximum limit of Zn in clean water = 5.0 ppm or mg dm^{-3}

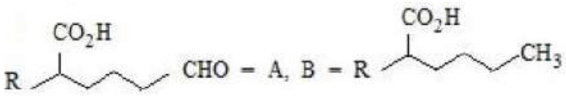
Maximum limit of NO_3^- in clean water = 50 ppm or mg dm^{-3}

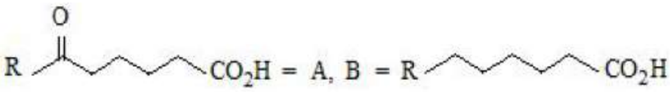
78. 

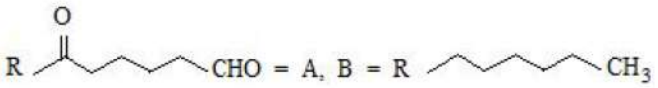
(R = alkyl)

'A' and 'B' in the above reactions are :

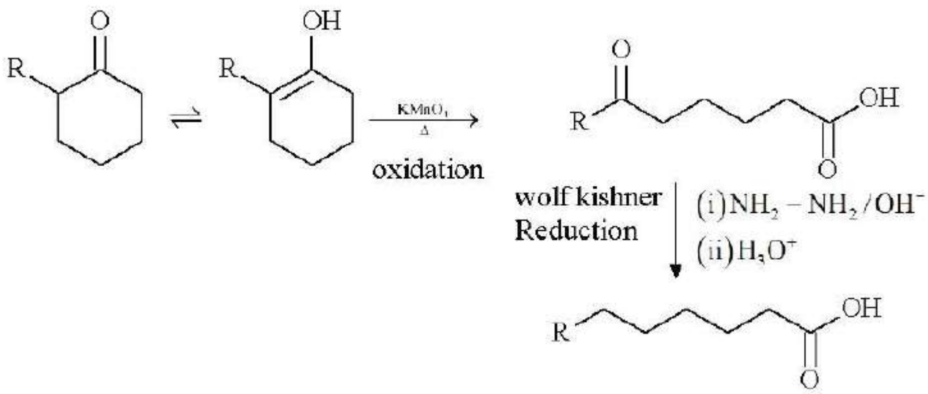
(1) 

(2) 

(3) 

(4) 

Sol. **3**



79. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R:

Assertion A : In the photoelectric effect electrons are ejected from the metal surface as soon as the beam of light of frequency greater than threshold frequency strikes the surface.

Reason R : When the photon of any energy strikes an electron in the atom transfer of energy from the photon to the electron takes place.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) A is correct but R is not correct

(2) A is not correct but R is correct

(3) Both A and R correct and R is the correct explanation of A

(4) Both A and R are correct but R is NOT the correct explanation of A

Sol. **1**

Assertion A is correct but Reason is not correct.

80. The complex that dissolves in water is

(1) $[Fe_3(OH)_2(OAc)_6]Cl$

(2) $Fe_4[Fe(CN)_6]_3$

(3) $K_3[Co(NO_2)_6]$

(4) $(NH_4)_3[As(Mo_3O_{10})_4]$

Sol. **1**

$Fe_4[Fe(CN)_6]_3$ Prussian Blue-water insoluble
$K_3[Co(NO_2)_6]$ very poorly water soluble
$(NH_4)_3$ $[As(MO_3O_{10})_4]$ water insoluble
ammonium arseno molybdate
$[Fe_3 (OH)_2(OAc)_6]$ Cl is water soluble.

SECTION - B

81. Solid fuel used in rocket is a mixture of Fe_2O_3 and Al (in ratio 1 : 2) the heat evolved (KJ) per gram of the mixture is _______ (Nearest integer)

Givne $\Delta H_f^\theta (Al_2O_3) = -1700$ KJ mol^{-1}

$\Delta H_f^\theta (Fe_2O_3) = -840$ KJ mol^{-1}

Sol. **4**

$Fe_2O_3 + 2Al \rightarrow Al_2O_3 + 2Fe$

$\Delta H_r = (\Delta H_f)\ Al_2O_3 - \Delta H_f°(Fe_2O_3)$

$= -1700 - (-840)$

$= -860$ kJ

Fe_2O_3 & Al $\rightarrow$ 1 : 2

Fe_2O_3 = 1 mole = (2 × 25 + 48)

= 112 + 48 = 160 gm

Al = 2 mole = 2 × 27 = 54 gm

Total mass = 160 + 54 = 214 gm

Heat evolved per gm = $\frac{-860}{214}$ kJ = –4.01 ≈ 4 kJ

82. $KClO_3 + 6FeSO_4 + 3H_2SO_4 \rightarrow KCl + 3Fe_2(SO_4)_3 + 3H_2O$

The above reaction was studied at 300 K by monitoring the concentration of $FeSO_4$ in which initial concentration was 10 M and after half an hour became 8.8 M. The rate of production of $Fe_2(SO_4)_3$ is______ $\times 10^{-6}$ mol L^{-1} s^{-1}

Sol. **333**

$$\frac{-\Delta FeSO_4}{\Delta t} = \frac{10-8.8}{30\times 60} = \frac{1.2}{1800}$$

From given equation :

$$-\frac{1}{6}\frac{\Delta FeSO_4}{\Delta t} = \frac{1}{3} \times \text{(Rate of production of } Fe_2(SO_4)_3\text{)}$$

Rate of production of $Fe_2(SO_4)_3$ = $\frac{3}{6}\times\frac{1.2}{1800}$

$= \frac{1}{3}\times 10^{-3}$

$= \frac{1000}{3}\times 10^{-6}$

$= 333.33 \times 10^{-6}$

83. 0.004 M K_2SO_4 solution is isotonic with 0.01 M glucose solution. Percentage dissociation of K_2SO_4 is________ (Nearest integer)

Sol. **75**

For isotonic solution

$(ic)_{glucose} = (ic)_{K_2SO_4}$

$0.01 = i\ (0.004)$

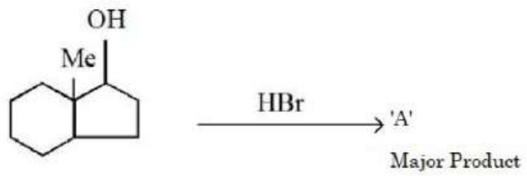
$i = \frac{0.01}{0.004} = \frac{10}{4} = \frac{5}{2}$

$1 + (n - 1)\ \alpha = \frac{5}{2}$

$1 + (3 - 1)\ \alpha = \frac{5}{2}$ $\quad (\because n = 3 \text{ for } K_2SO_4)$

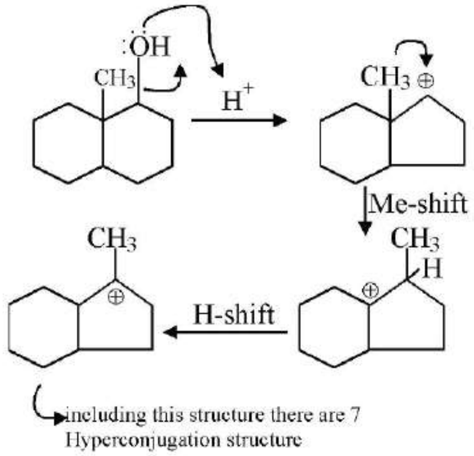
$2\alpha = \frac{3}{2}$

$\alpha = \frac{3}{4} \rightarrow 75\%$

84. 

The number of hyperconjugation structures involved to stabilize carbocation formed in the above reaction is______

Sol. **7**



85. A mixture of 1 mole of H_2O and 1 mole of CO is taken in a 10 litre container and heated to 725 K. At equilibrium 40% of water by mass reacts with carbon monoxide according to the equation : $CO(g)+H_2O(g) \rightleftharpoons CO_2(g)+H_2(g)$. The equilibrium constant $K_c \times 10^2$ for the reaction is_________ (Nearest integer)

Sol. **44**

	$CO(g)$ +	$H_2O(g) \rightleftharpoons$	$CO_2(g)$ +	$H_2(g)$
	1mole	1mole		
At equilibrium	1–0.4	1–0.4	0.4	0.4

$K_c = \frac{0.4 \times 0.4}{0.6 \times 0.6} = \frac{4}{9}$

$K_c \times 10^2 = \frac{4}{9} \times 100 = \frac{400}{9} = 44.44 \approx 44$

86. An atomic substance A of molar mass 12 g mol^{-1} has a cubic crystal structure with edge length of 300 pm. The no. of atoms present in one unit cell of A is ________ (Nearest integer)
Given the density of A is 3.0 g mL^{-1} and $N_A = 6.02\times10^{23}$ mol^{-1}

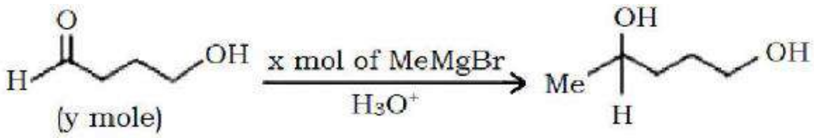
Sol. **4**

$$d = \frac{\frac{Z}{N_A}\times M}{a^3}$$

$$3 = \frac{Z}{6.02\times10^{23}}\times\frac{12}{(300\times10^{-10})^3}$$

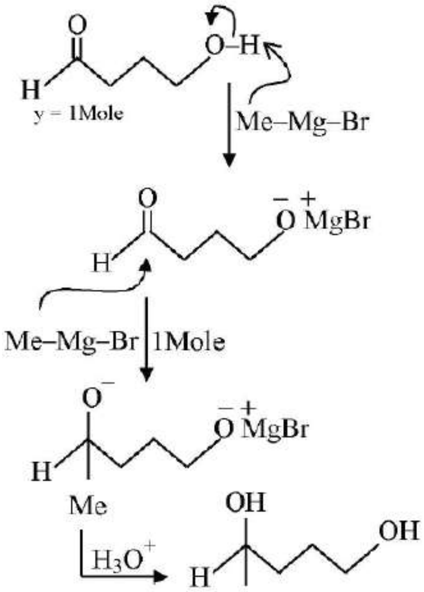
$$Z = \frac{3\times6.02\times27\times10^{6}\times10^{-30}\times10^{23}}{12}$$

$$= 40.635 \times 10^{-1} \quad = 4.0635 \approx 4$$

87. 

The ratio x/y on completion of the above reaction is_______

Sol. **2**



$\therefore$ x = 2 mole

$$\frac{x}{y} = \frac{2}{1} = 2$$

88. The ratio of spin-only magnetic moment values $\mu_{eff}[Cr(CN)_6]^{3-}/\mu_{eff}[Cr(H_2O)_6]^{3+}$ is_____

Sol. **1**

Spin magnetic moment of $[Cr(CN)_6]^{3-}(t_{2g}^3\, e_g^0)$

$$\mu_1 = \sqrt{3(3+2)} = \sqrt{15}BM$$

Spin magnetic moment of $[Cr(H_2O)_6]^{3+}(t_{2g}^3\, e_g^0)$

$$\mu_2 = \sqrt{3(3+2)} = \sqrt{15}BM$$

$$\frac{\mu_1}{\mu_2} = \frac{\sqrt{51}}{\sqrt{51}} = 1$$

89. In an electrochemical reaction of lead, at standard temperature, if $E^o_{(Pb^{2+}/Pb)} = m$ volt and $E^o_{(Pb^{4+}/Pb)} = n$ volt, then the value of $E^o_{(Pb^{2+}/Pb^{4+})}$ is given by m – xn. The value of x is ________ (Nearest integer)

Sol. **2**

$Pb^{2+} + 2e^- \rightarrow Pb$ $\quad E° = m \quad \Delta G^o_1 = -2Fm$

$Pb^{4+} + 4e^- \rightarrow Pb$ $\quad E° = n \quad \Delta G^o_2 = -4Fn$

$Pb^{2+} \rightarrow Pb^{4+} + 2e^-$ $\quad \Delta G^o_3 = \Delta G^o_1 - \Delta G^o_2$

$-2FE° = -2Fm + 4Fn$

$\boxed{E° = m - 2n}$

$\boxed{x = 2}$

90. A solution of sugar is obtained by mixing 200g of its 25% solution and 500g of its 40% solution (both by mass). The mass percentage of the resulting sugar solution is________(Nearest integer)

Sol. **36**

Solution (I) $\rightarrow$ Mass of sugar = $200 \times \frac{25}{100}$ = 50 gm

Mass of solution = 200 gm

Solution (II) $\rightarrow$ Mass of solution = 500 gm

Mass of sugar = $\frac{40}{100} \times 500$ = 200 gm

Final % w/w = $\frac{\text{Total mass of sugar}}{\text{Total mass of solution}} \times 100$

$= \frac{50 + 200}{200 + 500} \times 100 = \frac{250}{7}$

= 35.71% ≈ 36