CBSE NCERT Solutions for Class 11 Physics Chapter 4

Back of Chapter Questions

4.1. State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Solution:

Volume, mass, speed, density, number of moles, angular frequency are the scalar quantities.

Acceleration, velocity, displacement and angular velocity are the vector quantity.

A scalar quantity does have magnitude only. It does not have any direction associated with it. Volume, mass, speed, density, number of moles, and angular frequency are some of the examples of scalar quantities.

A vector quantity has magnitude as well as the direction associated with it. Acceleration, velocity, displacement, and angular velocity are some examples of a vector quantity.

4.2. Pick out the two scalar quantities in the following list: force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Solution:

Work and **current** are scalar quantities.

Work done is equals to the dot product of force and displacement. Since the two-quantity dot product is always a scalar, work is a physical scalar quantity.

Current does have a direction, but it does not follow the laws of vector addition but instead, get added like a vector. Hence, it is a scalar quantity.

4.3. Pick out the only vector quantity in the following list: Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Solution:

Impulse

The product of force and time gives impulse, which is a vector quantity.

4.4. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful: (a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Solution:

(a) Meaningful

It is only meaningful to add two scalar quantities if they both represent the same physical quantity.

(b) Not Meaningful

It is not meaningful to add a vector quantity with a scalar quantity.

(c) Meaningful

A scalar can be multiplied with a vector. For example, force is multiplied with time to give impulse.

(d) Meaningful

A scalar can be multiplied with another scalar having the same or different dimensions regardless of the physical quantity it represents.

(e) Meaningful

Adding two vector quantities is only relevant if both represent the same physical quantity.

(f) Meaningful

4.5. Read each statement below carefully and state with reasons, if it is true or false: (a) The magnitude of a vector is always a scalar, (b) each component of a vector is always a scalar, (c) the total path length is always equal to the magnitude of the displacement vector of a particle. (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time, (e) Three vectors not lying in a plane can never add up to give a null vector.

Solution:

(a) True

The magnitude of a vector is a scalar.

(b) False

Each component of a vector is also a vector.

(c) False

The total path length is a scalar quantity, whereas displacement is a vector quantity. Hence, the total path length is always greater than the magnitude of displacement. It becomes equal to the magnitude of displacement only when a particle is moving in a straight line.

(d) True

It is because of the fact that the total path length is always greater than or equal to the magnitude of displacement of a particle.

(e) True

Three vectors, which do not lie in a plane, cannot be represented by the sides of a triangle taken in the same order.

4.6. Establish the following vector inequalities geometrically or otherwise:

(a) $|a + b| < |a| + |b|$

(b) $|a + b| > ||a| - |b||$

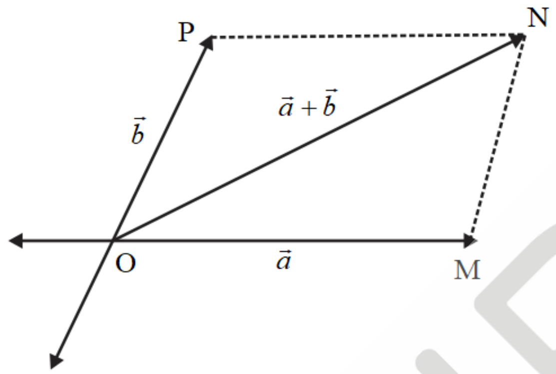
(c) $|a - b| < |a| + |b|$

(d) $|a - b| > ||a| - |b||$

When does the equality sign above apply?

Solution:

(a) The two vectors $\vec{a}$ and $\vec{b}$ can be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.



Here, we can write:

$|\overrightarrow{OM}| = |\vec{a}| \ldots$ (i)

$|\overrightarrow{MN}| = |\overrightarrow{OP}| = |\vec{b}| \ldots$ (ii)

$|\overrightarrow{ON}| = |\vec{a} + \vec{b}| \ldots$ (iii)

In a triangle, each side is smaller than the sum of the other two sides.

Therefore in ΔOMN we have

$|\vec{a} + \vec{b}| < |\vec{a}| + |\vec{b}| \ldots$ (iv)

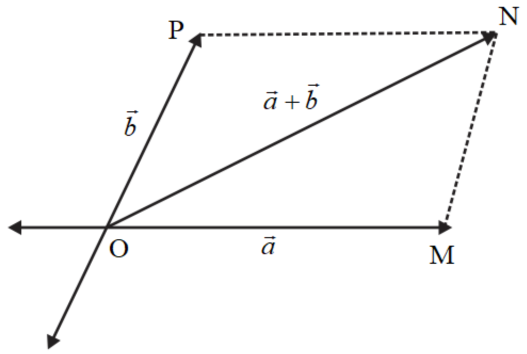
When the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then we can write

$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}| \ldots$ (v)

From equation (iv) & (v) we can write

$|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

(b) The two vectors $\vec{a}$ and $\vec{b}$ can be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.



$|\overrightarrow{OM}| = |\vec{a}| \ldots$ (i)

$|\overrightarrow{MN}| = |\overrightarrow{OP}| = |\vec{b}| \ldots$ (ii)

$|\overrightarrow{ON}| = |\vec{a} + \vec{b}| \ldots$ (iii)

In a triangle, the sum of the two sides is greater than the 3rd side.

Therefore, in ΔOMN, we have:

$ON + MN > OM$

$ON > OM - MN$

$\therefore OP = MN$

$|\overrightarrow{ON}| > |\overrightarrow{OM} - \overrightarrow{OP}|$

$|\vec{a} + \vec{b}| > \left||\vec{a}| - |\vec{b}|\right| \ldots$ (iv)

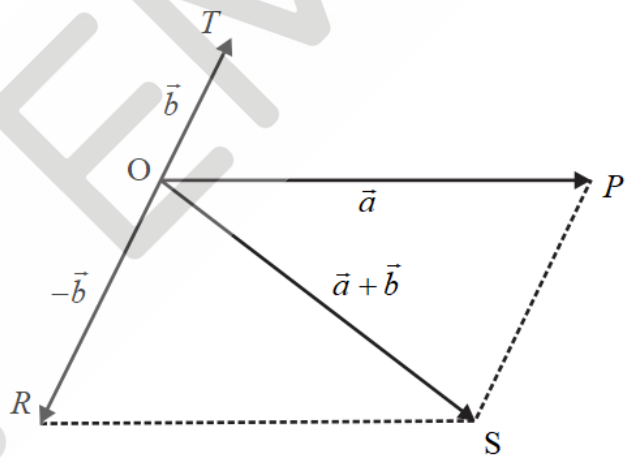
When two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then we can write

$|\vec{a} + \vec{b}| = |\vec{a}| - |\vec{b}| \ldots\ldots$ (v)

From equation (iv) & (v) zwe can write

$|\vec{a} + \vec{b}| \geq \left||\vec{a}| - |\vec{b}|\right|$

(c) The two vectors $\vec{a}$ and $\vec{b}$ can be represented by the adjacent sides of a parallelogram PORS, as shown in the given figure.



From the diagram,

$|\overrightarrow{OR}| = |\overrightarrow{PS}| = |\vec{b}| \ldots$ (i)

$|\overrightarrow{OP}| = |\vec{a}| \ldots$ (ii)

In a triangle, each side is smaller than the sum of the other two sides. Therefore, in ΔOPS, we have:

OS < OP + PS

$|\vec{a} - \vec{b}| < |\vec{a}| + |-\vec{b}|$

$|\vec{a} - \vec{b}| < |\vec{a}| + |\vec{b}| \dots$ (iii)

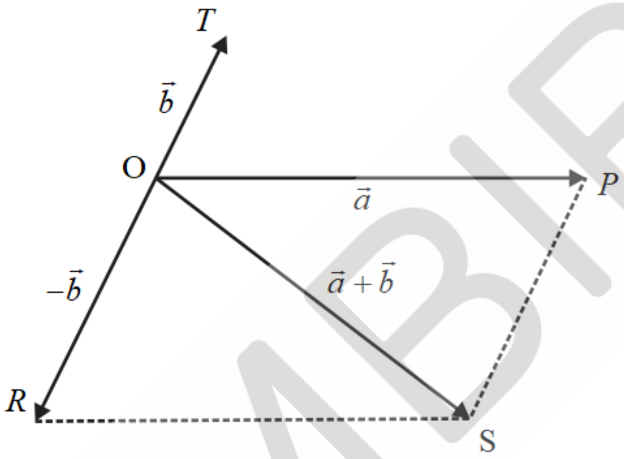
If two vectors act in a straight line but in opposite directions, then we can write:

$|\vec{a} - \vec{b}| = |\vec{a}| + |\vec{b}| \dots$ (iv)

Combining equation (iii) and (iv), we get:

$|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

(d) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram PORS, as shown in the given figure.



For the parallelogram, we can write

Therefore, in ΔOMN, we have:

$OS + PS > OP \dots$ (i)

$OS > OP - PS \dots$ (ii)

$|\vec{a} + \vec{b}| > \left||\vec{a}| - |\vec{b}|\right| \dots$ (iii)

If two vectors act in a straight line but in the opposite directions, then we can write

$|\vec{a} + \vec{b}| = \left||\vec{a}| - |\vec{b}|\right| \dots$ (iv)

Combining equation (iii) and (iv), we get:

$|\vec{a} + \vec{b}| \geq \left||\vec{a}| - |\vec{b}|\right|$

4.7. Given $a + b + c + d = 0$, which of the following statements are correct:

(a) a, b, c, and d must each be a null vector,

(b) The magnitude of $(a + c)$ equals the magnitude of $(b + d)$,

(c) The magnitude of a can never be greater than the sum of the magnitudes of b, c, and d,

(d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are

Solution:

(a) Incorrect

In order to make a + b + c + d = 0, we need to for a, b, c and d to form a quadrilateral. It is not necessary to have all the four given vectors to be null vectors.

(b) Correct

a + b + c + d = 0

a + c = − (b + d)

Taking modulus on both the sides, we get:

|a + c| = |− (b + d)| = |b + d|

Hence, the magnitude of (a + c) is the same as the magnitude of (b + d).

(c) Correct

a + b + c + d = 0

a = (b + c + d)

Taking modulus both sides, we get:

| a | = | b + c + d | … (i)

Equation (i) shows that the magnitude of a is equal to or less than the sum of the magnitudes of b, c, and d.

Hence, the magnitude of a vector can never be greater than the sum of the magnitudes of b, c, and d.

(d) Correct

For a + b + c + d = 0

a + (b + c) + d = 0

The resultant sum of the three vectors a, (b + c), and d can be zero only if (b + c) lie in a plane of a and d and these three vectors are represented by the three sides of a triangle.

If a and d are collinear, then it implies that the vector (b + c) is in the line of a and d. This implication holds only then the vector sum of all the vectors will be zero.

4.8. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 4.20. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?

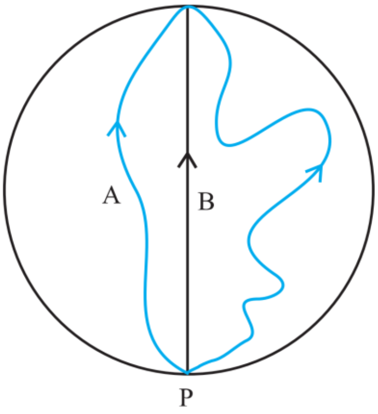


Fig 4.20

Solution:

Displacement is given by the minimum distance between a particle's initial and final positions. In the given case, all the girls start from point P and reach point Q. The magnitudes of their displacements will be equal to the diameter of the ground.

The radius of the ground $= 200$ m

Diameter of the ground $= 2 \times 200 = 400$ m

Hence, the magnitude of the displacement for each girl is 400 m. This is equal to the actual length of the path skated by girl B.

4.9. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. 4.21. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?

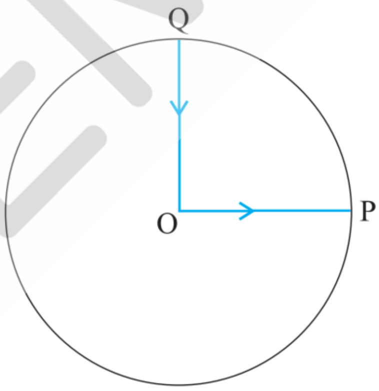


Fig. 4.21

Solution:

(a) Displacement is given by the minimum distance between the initial and final positions of a body. In the given case, the cyclist comes to the starting point after cycling for 10 minutes. Hence, his net displacement is zero.

(b) Average velocity is given by the relation:

Average velocity = Net Displacement / Total Time

Since the cyclist's net displacement is zero, his average velocity will also be zero.

(c) Average speed of the cyclist is given by the relation:

Average speed = Total Path Length / Total Time

Total path length = $OP + PQ + QO$

$= 1 + 1/4\,(2\pi \times 1) + 1$

$= 2 + 1/2\,\pi = 3.570$ km

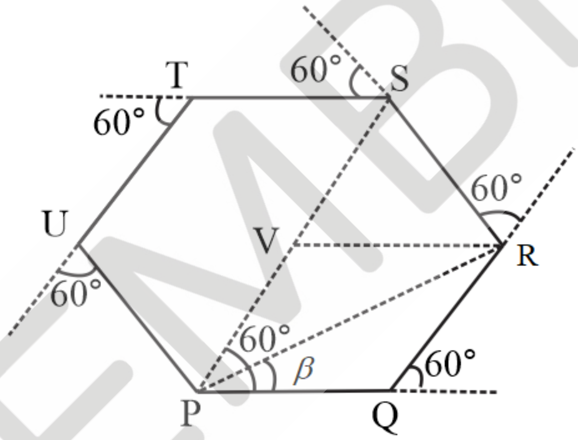
Time taken = 10 min $= 10/60 = 1/6$ h

$\therefore$Average speed $= \frac{3.570}{1/6} = 21.42$ km/h

4.10. On an open ground, a motorist follows a track that turns to his left by an angle of 600 after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Solution:

The path followed by the motorist is a regular hexagon with side 500 m, as shown in the given figure



Let the motorist start from point P.

The motorist takes the third turn at S.

$\therefore$Magnitude of displacement $= \text{PS} = \text{PV} + \text{VS} = 500 + 500 = 1000$ m

Total path length $= \text{PQ} + \text{QR} + \text{RS} = 500 + 500 + 500 = 1500$ m

The motorist takes the sixth turn at point P, which is the starting point.

$\therefore$Magnitude of displacement $= 0$

Total path length $= \text{PQ} + \text{QR} + \text{RS} + \text{ST} + \text{TU} + \text{UP}$

$= 500 + 500 + 500 + 500 + 500 + 500 = 3000$ m

The motorist takes the eight turn at point R

$\therefore$ The magnitude of displacement $=$ PR

$\sqrt{PQ^2 + QR^2 + 2(PQ)(QR)\cos 60^o}$

$$\sqrt{500^2 + 500^2 + 2(500)(500)\left(\frac{1}{2}\right)} = 866.03 \text{ m}$$

$$\beta = \tan^{-1}\left(\frac{500 \sin 60^o}{500 + 500 \cos 60^o}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^o$$

Therefore, the magnitude of displacement is 866.03 m at an angle of 30^o with PR.

Total path length = Circumference of the hexagon + PQ + QR

$= 6 \times 500 + 500 + 500 = 4000$ m

The magnitude of displacement and the total path length corresponding to the required turns is shown in the given table:

Turn	**The magnitude of displacement (m)**	**Total path length (m)**
Third	1000	1500
Sixth	0	3000
Eighth	866.03; 30^o	4000

4.11. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

Solution:

(a) Total distance travelled = 23 km

Total time taken = 28 min = 28/60 h

∴ Average speed = Total Distance/ Total Time

$= 23 / (28/60) = 49.29$ km/h

(b) Displacement of the car = 10 km

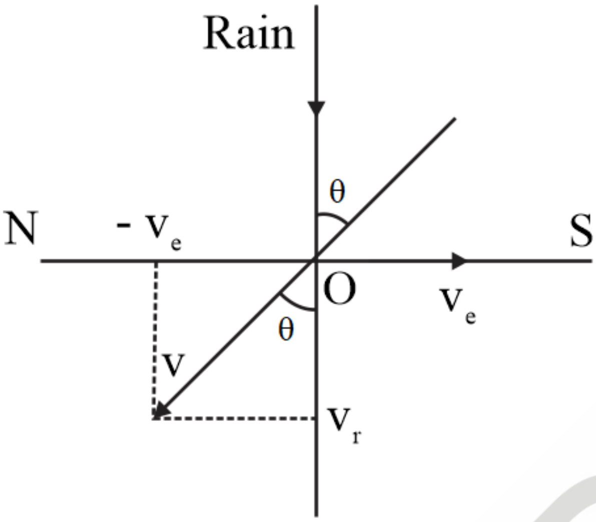
∴ Average velocity $= 10 / (28/60) = 21.43$ km/h

Therefore, the average speed and average velocity are not equal.

4.12. Rain is falling vertically with a speed of 30 m s^{-1}. A woman rides a bicycle with a speed of 10 m s^{-1} in the north to south direction. What is the direction in which she should hold her umbrella?

Solution:

The described situation is shown in the given figure.



Here,

v_c = Velocity of the cyclist

v_r = Velocity of falling rain

In order to protect herself from the rain, the woman must hold her umbrella in the direction of the relative velocity (v) of the rain with respect to the woman.

$v = v_r + (-v_c)$

$= 30 + (-10) = 20$ m/s

$\tan\theta = \frac{v_c}{v_r} = \frac{10}{30}$

$\theta = \tan^{-1}\frac{1}{3}$

$= \tan^{-1}(0.3333) = 18°$

Hence, the woman must hold the umbrella toward the south, at an angle of nearly 18° with the vertical.

4.13. A man can swim with a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Solution:

Speed of the man, $v_m = 4$ km/h

Width of the river = 1 km

Time taken to cross the river $= \frac{\text{width of the river}}{\text{speed of the river}} = \frac{1}{4}$ h = 15 minutes

Speed of the river, $v_r = 3$ km/h

Distance covered with the flow of the river $= v_r \times t$

$= 3 \times \frac{1}{4} = 3/4$ km

$$= \frac{3}{4} \times 1000 \ = \ 750 \text{ m}$$

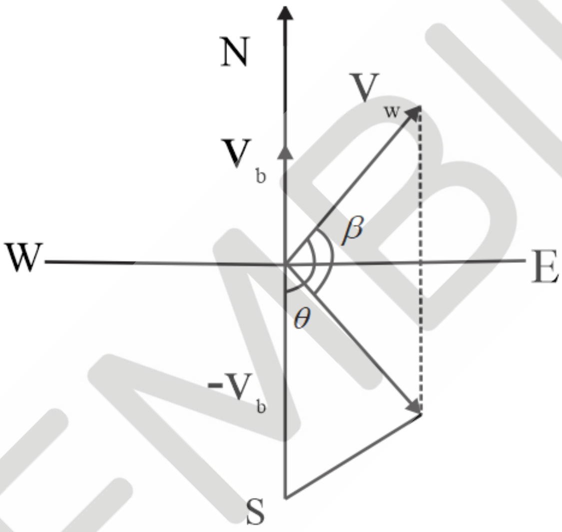
4.14. In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

Solution:

The velocity of the boat, $v_b = 51$ km/h

The velocity of the wind, $v_w = 72$ km/h

The flag is fluttering in the north-east direction. It shows that the wind is blowing toward the north-east direction. When the ship begins sailing toward the north, the flag will move along the direction of the relative velocity (v_{wb}) of the wind with respect to the boat.



The angle between v_w and $(-v_b)$ = $90^o + 45^o$

$$\tan\beta = \frac{51\sin(90+45)}{72 + 51\cos(90+45)} = \frac{\frac{51}{\sqrt{2}}}{71 - \frac{51}{\sqrt{2}}} = 1.06$$

$\therefore \beta = \tan^{-1} 1.06 \approx 46^o$

Angle with respect to the east direction = $46^o - 45^o = 1^o$

Hence, the flag will flutter almost due east.

4.15. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall?

Solution:

Speed of the ball, u = 40 m/s

Maximum height, h = 25 m

In projectile motion, the maximum height reached by a body projected at an angle θ, is given by the relation:

$$h = \frac{u^2 \sin^2\theta}{2g}$$

$$25 = \frac{40^2 \sin^2\theta}{2 \times 9.8}$$

$$\sin^2\theta = 0.306$$

$$\sin\theta = 0.553$$

$$\therefore \theta = \sin^{-1} 0.553 = 33.60^o$$

$$\text{Horizontal range, } R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{40^2 \times \sin 2 \times 33.60}{9.8}$$

$$= \frac{1600 \times \sin 67.2}{9.8} = 150.53 \text{ m}$$

4.16. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Solution:

Maximum horizontal distance, R = 100 m

Maximum range is obtained at an angle, $\theta = 45^o$.

The horizontal range for a projection velocity u is given by the relation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$100 = \frac{u^2}{g \sin 90^o}$$

$$\frac{u^2}{g} = 100 \quad .... \text{(i)}$$

When thrown vertically upward, the ball will achieve the maximum height. At the maximum height, the velocity of object is zero.

Acceleration, $a = -g$

Using the third equation of motion:

$$v^2 - u^2 = -2gH$$

$$H = \frac{1}{2} \times \frac{u^2}{g} = \frac{1}{2} \times 100 = 50 \text{ m}$$

4.17. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Solution:

Length of the string, $l = 80 \text{ cm} = 0.8 \text{ m}$

Number of revolutions $= 14$

Time taken $= 25 \text{ s}$

Frequency, $f = \frac{\text{Number of revolutions}}{\text{time taken}} = \frac{14}{25} \text{ Hz}$

Angular frequency, $\omega = 2\pi f$

$= 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25} \text{ rad s}^{-1}$

Centripetal acceleration, $a_c = \omega^2 r$

$= \left(\frac{88}{25}\right)^2 \times 0.8 = 9.91 \text{ m/s}^2$

The direction of centripetal acceleration is along the string (toward the centre) at all points.

4.18. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Solution:

Radius of the loop, $r = 1 \text{ km} = 1000 \text{ m}$

Speed of the aircraft, $v = 900 \text{ km/h} = 900 \times 5/18 = 250 \text{ m/s}$

Centripetal acceleration, $a_c = \frac{v^2}{r} = (250)^2/1000 = 62.5 \text{ m/s}^2$

Acceleration due to gravity, $g = 9.8 \text{ m/s}^2$

$a_c/g = 62.5/9.8 = 6.38$

$a_c = 6.38g$

The centripetal acceleration is 6.38 times the gravitational acceleration.

4.19. Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector

Solution:

(a) False,

The net acceleration of a particle in a circular motion is not always directed at the centre along the radius of the circle. It only occurs in the case of uniform circular motion.

(b) True

A particle appears to be moving tangentially to the circular path at a point on a circular path. Therefore, the particle's velocity vector is always at a point along the tangent.

(c) True

The direction of the acceleration vector points towards the centre of the circle in a uniform circular motion. However, it constantly changes with time. Hence, the average of these vectors over one cycle is a null vector.

4.20. The position of a particle is given by

$r = 3.0t\,\hat{\imath} - 2.0t^2\hat{\jmath} + 4.0\,\hat{k}\text{ m}$

Where t is in seconds and the coefficients have the proper units for r to be in meters.

(a) Find the v and a of the particle?

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s?

Solution:

(a) The position of the particle is given by

$r = 3.0t\,\hat{\imath} - 2.0t^2\hat{\jmath} + 4.0\,\hat{k}\text{ m}$

The velocity vector can be calculated as

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0t\,\hat{\imath} - 2.0t^2\hat{\jmath} + 4.0\,\hat{k}) = 3.0\,\hat{\imath} - 4.0t\,\hat{\jmath}\text{ m/s}$$

The acceleration of particle can be calculated as

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3.0\,\hat{\imath} - 4.0t\,\hat{\jmath}) = -4.0\,\hat{\jmath}\text{ m/s}^2$$

(b) We have the velocity vector, $\vec{v} = 3.0\,\hat{\imath} - 4.0t\,\hat{\jmath}$

At $t = 2.0$ s

$\vec{v} = 3.0\,\hat{\imath} - 8.0\,\hat{\jmath}$

The magnitude of the velocity vector is

$|\vec{v}| = \sqrt{3^2 + 8^2} = \sqrt{73} = 8.54\text{ m/s}$

Direction, $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1} -\frac{8}{3}$

$= -\tan^{-1} 2.667 = -69.45^{\text{o}}$

The negative sign indicates that the direction of velocity is below the *x*-axis.

4.21. A particle starts from the origin at $t = 0$ s with a velocity of $10.0\,\hat{\jmath}$ m/s and moves in the x-y plane with a constant acceleration of $8.0\,\hat{\imath} + 2.0\,\hat{\jmath}\text{ m s}^{-2}$. (a) At what time is the x- coordinate of the particle 16 m? What is the y-coordinate of the particle at that time? (b) What is the speed of the particle at the time?

Solution:

(a) $x = 16$ m

$\vec{v} = 10.0\,\hat{\jmath}\text{ m/s}$

$\vec{a} = 8.0\,\hat{\imath} + 2.0\,\hat{\jmath}\ \text{m s}^{-2}$

From the equation of motion in the x-direction

$16 = 0 \times t + \frac{1}{2} \times 8\,t^2$

$\Rightarrow 4t^2 = 16 \Rightarrow t^2 = 4 \Rightarrow t = 2\ \text{s}$

In y-direction

$y = 10 \times t + \frac{1}{2} \times 2 \times t^2$

Put $t = 2$ s

$y = 10 \times 2 + \frac{1}{2} \times 2 \times 2^2 = 24\ \text{m}$

(b) Speed of particle after 2 sec

$\vec{v} = 10.0\,\hat{\jmath} + (8.0\,\hat{\imath} + 2.0\,\hat{\jmath}) \times 2 = 16.0\,\hat{\imath} + 14.0\,\hat{\jmath}\ \text{m/s}$

4.22. $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors along x- and y-axis respectively. What is the magnitude and direction of the vectors $\hat{\imath} + \hat{\jmath}$, and $\hat{\imath} - \hat{\jmath}$? What are the components of a vector $\mathbf{A} = 2\hat{\imath} + 3\hat{\jmath}$ along the directions of $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - \hat{\jmath}$? [You may use graphical method]

Solution:

The magnitude of the vector $\hat{\imath} + \hat{\jmath}$ is,

$\sqrt{1^2 + 1^2} = \sqrt{2}$

The angle made by the vector with the x-axis,

$\tan\theta = \frac{y}{x} = 1$

$\Rightarrow \theta = 45^{\circ}$

The magnitude of the vector $\hat{\imath} - \hat{\jmath}$ is,

$\sqrt{1^2 + (-1)^2} = \sqrt{2}$

The angle made by the vector with the x-axis,

$\tan\theta = \frac{y}{x} = -1$

$\Rightarrow \theta = -45^{\circ}$

The components of a vector $\mathbf{A}$ along the vector $\hat{\imath} + \hat{\jmath}$ is

$$\frac{(2\hat{\imath} + 3\hat{\jmath}).(\hat{\imath} + \hat{\jmath})}{|\hat{\imath} + \hat{\jmath}|} = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

The components of a vector $\mathbf{A}$ along the vector $\hat{\imath} - \hat{\jmath}$ is

$$\frac{(2\hat{\imath} + 3\hat{\jmath}).(\hat{\imath} - \hat{\jmath})}{|\hat{\imath} - \hat{\jmath}|} = \frac{2-3}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

4.23. For any arbitrary motion in space, which of the following relations are true:

(a) $v_{average} = \left(\frac{1}{2}\right)(v(t_1) + v(t_2))$

(b) $v_{average} = [r(t_2) - r(t_1)]/(t_2 - t_1)$

(c) $v(t) = v(0) + at$

(d) $r(t) = r(0) + v(0)t + (1/2)at^2$

(e) $a_{average} = [v(t_2) - v(t_1)]/(t_2 - t_1)$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

Solution: (b) and (e)

(a) The particle's movement is given to be arbitrary. Thus, this equation can not give the average particle velocity.

(b) The average velocity for the arbitrary motion of the particle can be represented by this equation.

(c) The motion of the particle is arbitrary. Hence, acceleration of the particle may be non-uniform. This equation of motion represents a uniformly accelerated motion. Hence, this equation cannot represent any arbitrary motion of the particle in space.

(d) The motion of the particle is arbitrary. Hence, acceleration of the particle may be non-uniform. This equation of motion represents a uniformly accelerated motion. Hence, this equation cannot represent any arbitrary motion of the particle in space.

(e) The given equation for average acceleration in case of uniformly accelerated motion. Hence, the average velocity of an arbitrary motion of the particle can be represented by this equation.

4.24. Read each statement below carefully and state, with reasons and examples, if it is true or false:

A scalar quantity is one that

(a) is conserved in a process

(b) can never take negative values

(c) must be dimensionless

(d) does not vary from one point to another in space

(e) has the same value for observers with different orientations of axes

Solution:

(a) False

Despite being a scalar quantity, kinetic energy is not conserved in inelastic collisions.

(b) False

Despite being a scalar quantity, temperature can take negative values.

(c) False

The total path length is a scalar quantity. Yet it has the dimension of length.

(d) False

A scalar quantity like gravity potential can differ in space from point to point.

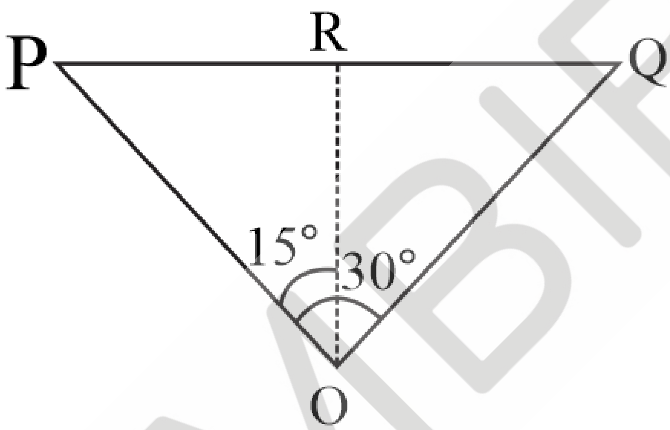
(e) True

The value of a scalar does not vary for observers with different orientations of axes. The mass of a person remains same no matter from which direction you observe.

4.25. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30^o, what is the speed of the aircraft?

Solution:

The positions of the observer (O) and the aircraft are shown in the given figure.



Height of the aircraft from the ground, $OR = 3400$ m

The angle subtended between the positions, $\angle POQ = 30^o$

Time $= 10$ s

In ΔPRO:

$\tan 15^o = PR/OR$

$PR = OR \tan 15^o$

$= 3400 \times \tan 15^o$

ΔPRO is similar to ΔRQO

$\therefore PR = RQ$

$PQ = PR + RQ$

$= 2PR = 2 \times 3400 \tan 15^o$

$= 6800 \times 0.268 = 1822.4$ m

$\therefore$Speed of the aircraft $= 1822.4/10 = 182.24$ m/s

Additional Exercises

4.26. A vector has magnitude and direction. Does it have a location in space? Can it vary with time? Will two equal vectors a and b at different locations in space necessarily have identical physical effects? Give examples in support of your answer.

Solution:

Does it have a location in space? **No;**

Generally speaking, a vector has no definite locations in space. This is because a vector remains invariant when displaced in such a way that its magnitude and direction remain the same. However, a position vector has a definite location in space.

Can it vary with time? **Yes;**

A vector can vary with time. For example, the displacement vector of a particle moving with a certain velocity varies with time.

Will two equal vectors a and b at different locations in space necessarily have identical physical effects? **No**

Two equal vectors located at different locations in space need not produce the same physical effect. For example, two equal forces acting on an object at different points can cause the body to rotate, but their combination cannot produce an equal turning effect.

4.27. A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector?

Solution:

(i) No, it Does not mean that anything that has magnitude and direction is necessarily a vector.

A physical quantity having both magnitude and direction need not be considered as a vector quantity. For example, despite having magnitude and direction, the current is not a vector quantity, it is a scalar quantity. The essential requirement for a physical quantity to be considered a vector is that it should follow the law of vector addition.

(ii) The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector? **No**

Generally speaking, a body's rotation around an axis is not a vector quantity because it does not follow the law of vector addition. However, rotation by a certain small angle follows the law of vector addition and is therefore considered a vector.

4.28. Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere? Explain.

Solution:

(a) No, we cannot associate a vector with the length of a wire bent into a loop.

(b) Yes, we can associate an area vector with a plane area. The direction of this vector is normal, inward or outward to the plane area.

(c) No, we cannot associate a vector with the volume of a sphere. However, an area vector can be associated with the area of a sphere.

4.29. A bullet fired at an angle of 30^o with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the muzzle speed to the fixed, and neglect air resistance.

Solution:

Given:

Range, $R = 3\text{ km} = 3000\text{ m}$

The angle of projection, $\theta = 30^o$

Acceleration due to gravity, $g = 9.8\text{ m/s}^2$

We know,

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow 3000 = \frac{u^2 \sin 60^o}{9.8}$$

$$\Rightarrow \frac{u^2}{g} = 2000\sqrt{3}$$

The maximum range for the projectile,

$$R_{\max} = \left(\frac{u^2}{g}\right) = 2000\sqrt{3}\text{ m} = 3464.1\text{ m}$$

Since the maximum range of the bullet is 3.46 km, it will never hit the target 5.0 km away.

4.30. A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take $g = 10\text{ m s}^{-2}$).

Solution:

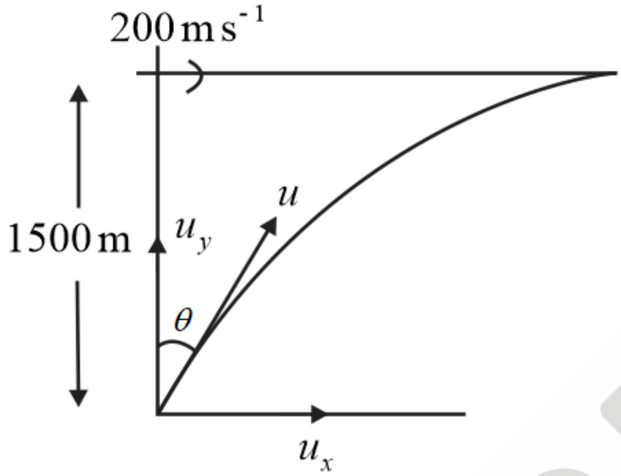
Given:

Height of fighter plane $= 1.5\text{ km} = 1500\text{ m}$

Speed of plane $v = 720\text{ km/h} = 200\text{ m/s}$

Muzzle speed $u = 600\text{ m s}^{-1}$

Let the angle made by the shell with vertical is θ as shown in the figure.



Let the time taken by the shell to hit the plane is t.

Then,

Horizontal distance travelled by the shell in t s = Horizontal distance travelled by plane in t s

$u \sin\theta \times t = v \times t$

$\Rightarrow 600 \sin\theta = 200 \Rightarrow \sin\theta = \frac{1}{3}$

In order to avoid being hit, the plane should fly above the maximum height achieved by the shell.

$$h = \frac{u^2 \cos^2\theta}{2g} = \frac{u^2(1-\sin^2\theta)}{2g} = \frac{(600)^2\left(1-\left(\frac{1}{3}\right)^2\right)}{2 \times 10} = 16000 \text{ m or } 16 \text{ km}$$

4.31. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Solution:

Given:

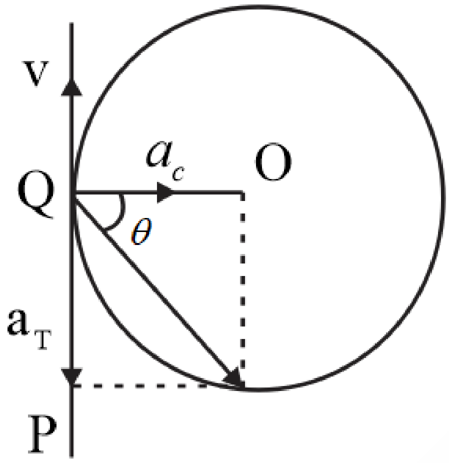
Speed of cyclist $v = 27 \text{ km/h} = 7.5 \text{ m/s}$

The radius of circular turn $R = 80$ m

Tangential acceleration $a_t = -0.50 \text{ m/s}^2$

The centripetal acceleration $a_c = \frac{v^2}{R} = \frac{(7.5)^2}{80} = 0.703 \text{ m/s}^2$

Since the centripetal and tangential acceleration are perpendicular to each other.



The magnitude of acceleration,

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(0.703)^2 + (-0.5)^2} = 0.863 \text{ m/s}^2$$

If the angle of the net acceleration with velocity is θ then

$$\tan\theta = \frac{a_c}{a_t} = \frac{0.703}{0.50} = 1.4$$

4.32. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

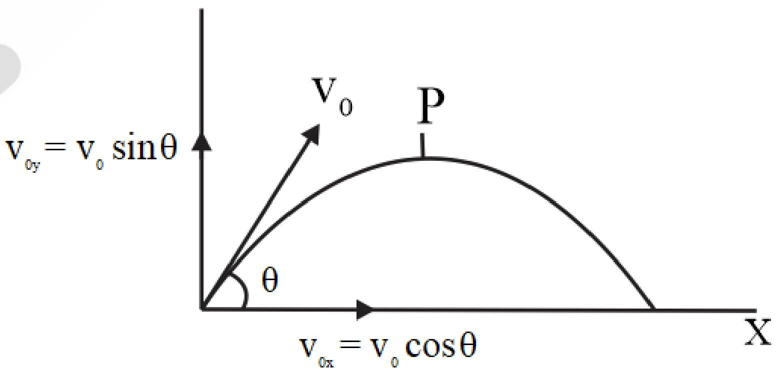
(b) Shows that the projection angle θ0 for a projectile launched from the origin is given by

$$\theta(t) = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

Solution:

(a) Let v_{0x} and v_{0y} are initial horizontal and vertical component of velocity and v_x and v_y are the horizontal and vertical component of velocity at point P.



The time taken by projectile to reach point P is t, then

$v_x = v_{0x}$

$v_y = v_{0y} - gt$

$\therefore \tan\theta = \frac{v_y}{v_x} = \frac{v_{0y} - gt}{v_{0x}}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$

(b) The maximum vertical height can be written as,

$h_m = \frac{v^2 \sin^2\theta}{2g} \dots$ (i)

And the horizontal range, $R = \frac{v^2 \sin 2\theta}{g} \dots$ (ii)

By dividing equation (i) and (ii)

$\frac{h_m}{R} = \frac{\sin^2\theta}{2\sin 2\theta}$

$\Rightarrow \frac{h_m}{R} = \frac{\sin^2\theta}{2 \times 2\sin\theta\cos\theta}$

$\Rightarrow \frac{h_m}{R} = \frac{\tan\theta}{4}$

$\Rightarrow \tan\theta = \frac{4h_m}{R}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{4h_m}{R}\right)$